CHAPTER 11
PROBLEM 11.1

The motion of a particle is defined by the relation \( x = 1.5t^4 - 30t^2 + 5t + 10 \), where \( x \) and \( t \) are expressed in meters and seconds, respectively. Determine the position, the velocity, and the acceleration of the particle when \( t = 4 \) s.

SOLUTION

Given:

\[
\begin{align*}
x &= 1.5t^4 - 30t^2 + 5t + 10 \\
v &= \frac{dx}{dt} = 6t^3 - 60t + 5 \\
a &= \frac{dv}{dt} = 18t^2 - 60
\end{align*}
\]

Evaluate expressions at \( t = 4 \) s.

\[
\begin{align*}
x &= 1.5(4)^4 - 30(4)^2 + 5(4) + 10 = -66 \text{ m} \\
v &= 6(4)^3 - 60(4) + 5 = 149 \text{ m/s} \\
a &= 18(4)^2 - 60 = 228 \text{ m/s}^2
\end{align*}
\]

\[\text{x = -66.0 m} \quad \text{v = 149.0 m/s} \quad \text{a = 228.0 m/s}^2\]
PROBLEM 11.2

The motion of a particle is defined by the relation \( x = 12t^3 - 18t^2 + 2t + 5 \), where \( x \) and \( t \) are expressed in meters and seconds, respectively. Determine the position and the velocity when the acceleration of the particle is equal to zero.

SOLUTION

Given:

\[
\begin{align*}
x(t) &= 12t^3 - 18t^2 + 2t + 5 \\
v(t) &= \frac{dx}{dt} = 36t^2 - 36t + 2 \\
a(t) &= \frac{dv}{dt} = 72t - 36
\end{align*}
\]

Find the time for \( a = 0 \).

\[
72t - 36 = 0 \Rightarrow t = 0.5 \text{ s}
\]

Substitute into above expressions.

\[
\begin{align*}
x(t) &= 12(0.5)^3 - 18(0.5)^2 + 2(0.5) + 5 = 3 \\
v(t) &= 36(0.5)^2 - 36(0.5) + 2 = -7 \text{ m/s}
\end{align*}
\]

\( x = 3.00 \text{ m} \)

\( v = -7.00 \text{ m/s} \)
**PROBLEM 11.3**

The motion of a particle is defined by the relation $x = \frac{5}{3}t^3 - \frac{5}{2}t^2 - 30t + 8$, where $x$ and $t$ are expressed in meters and seconds, respectively. Determine the time, the position, and the acceleration when $v = 0$.

**SOLUTION**

We have

\[ x = \frac{5}{3}t^3 - \frac{5}{2}t^2 - 30t + 8 \]

Then

\[ v = \frac{dx}{dt} = 5t^2 - 5t - 30 \]

and

\[ a = \frac{dv}{dt} = 10t - 5 \]

When $v = 0$:

\[ 5t^2 - 5t - 30 = 5(t^2 - t - 6) = 0 \]

or

\[ t = 3 \text{ s} \quad \text{and} \quad t = -2 \text{ s} \quad (\text{Reject}) \]

\[ t = 3.00 \text{ s} \]

At $t = 3$ s:

\[ x_3 = \frac{5}{3}(3)^3 - \frac{5}{2}(3)^2 - 30(3) + 8 \]

or

\[ x_3 = -59.5 \text{ m} \]

\[ a_3 = 10(3) - 5 \]

or

\[ a_3 = 25.0 \text{ m/s}^2 \]
PROBLEM 11.4

The motion of a particle is defined by the relation \( x = 6t^2 - 8 + 40\cos\pi t \), where \( x \) and \( t \) are expressed in meters and seconds, respectively. Determine the position, the velocity, and the acceleration when \( t = 6 \) s.

**SOLUTION**

We have \( x = 6t^2 - 8 + 40\cos\pi t \)

Then \( v = \frac{dx}{dt} = 12t - 40\pi\sin\pi t \)

and \( a = \frac{dv}{dt} = 12 - 40\pi^2\cos\pi t \)

At \( t = 6 \) s:

\[ x_6 = 6(6)^2 - 8 + 40\cos(6\pi) \]  
  or  \( x_6 = 248 \) m

\[ v_6 = 12(6) - 40\pi\sin(6\pi) \]  
  or  \( v_6 = 72.0 \) m/s

\[ a_6 = 12 - 40\pi^2\cos(6\pi) \]  
  or  \( a_6 = -383 \) m/s^2
PROBLEM 11.5

The motion of a particle is defined by the relation \( x = 6t^4 - 2t^3 - 12t^2 + 3t + 3 \), where \( x \) and \( t \) are expressed in meters and seconds, respectively. Determine the time, the position, and the velocity when \( a = 0 \).

SOLUTION

We have
\[
x = 6t^4 - 2t^3 - 12t^2 + 3t + 3
\]
Then
\[
v = \frac{dx}{dt} = 24t^3 - 6t^2 - 24t + 3
\]
and
\[
a = \frac{dv}{dt} = 72t^2 - 12t - 24
\]
When \( a = 0 \):
\[
72t^2 - 12t - 24 = 12(6t^2 - t - 2) = 0
\]
or
\[
(3t - 2)(2t + 1) = 0
\]
or
\[
t = \frac{2}{3} \text{ s and } t = -\frac{1}{2} \text{ s (Reject)}
\]
\[
t = 0.667 \text{ s} \blacktriangle
\]
At \( t = \frac{2}{3} \) s:
\[
x_{\frac{2}{3}} = 6\left(\frac{2}{3}\right)^4 - 2\left(\frac{2}{3}\right)^3 - 12\left(\frac{2}{3}\right)^2 + 3\left(\frac{2}{3}\right) + 3 \quad \text{or} \quad x_{\frac{2}{3}} = 0.259 \text{ m} \blacktriangle
\]
\[
v_{\frac{2}{3}} = 24\left(\frac{2}{3}\right)^3 - 6\left(\frac{2}{3}\right)^2 - 24\left(\frac{2}{3}\right) + 3 \quad \text{or} \quad v_{\frac{2}{3}} = -8.56 \text{ m/s} \blacktriangle
\]
PROBLEM 11.6

The motion of a particle is defined by the relation \( x = 2t^3 - 15t^2 + 24t + 4 \), where \( x \) is expressed in meters and \( t \) in seconds. Determine (a) when the velocity is zero, (b) the position and the total distance traveled when the acceleration is zero.

SOLUTION

\[
x = 2t^3 - 15t^2 + 24t + 4
\]

\[
v = \frac{dx}{dt} = 6t^2 - 30t + 24
\]

\[
a = \frac{dv}{dt} = 12t - 30
\]

(a) \( v = 0 \) when 

\[6t^2 - 30t + 24 = 0\]

\[6(t - 1)(t - 4) = 0 \quad t = 1.000 \text{ s} \quad \text{or} \quad t = 4.00 \text{ s}\]

(b) \( a = 0 \) when 

\[12t - 30 = 0 \quad t = 2.5 \text{ s}\]

For \( t = 2.5 \text{ s} \):

\[x_{2.5} = 2(2.5)^3 - 15(2.5)^2 + 24(2.5) + 4 \]

\[x_{2.5} = +1.500 \text{ m}\]

To find total distance traveled, we note that 

\( v = 0 \) when \( t = 1 \text{ s} \):

\[x_1 = 2(1)^3 - 15(1)^2 + 24(1) + 4 \]

\[x_1 = +15 \text{ m}\]

For \( t = 0 \),

\[x_0 = +4 \text{ m}\]

Distance traveled

From \( t = 0 \) to \( t = 1 \text{ s} \):

\[x_1 - x_0 = 15 - 4 = 11 \text{ m} \rightarrow\]

From \( t = 1 \text{ s} \) to \( t = 2.5 \text{ s} \):

\[x_{2.5} - x_1 = 1.5 - 15 = 13.5 \text{ m} \leftarrow\]

Total distance traveled \( = 11 \text{ m} + 13.5 \text{ m} = 24.5 \text{ m}\)
PROBLEM 11.7

The motion of a particle is defined by the relation \( x = t^3 - 6t^2 - 36t - 40 \), where \( x \) and \( t \) are expressed in meters and seconds, respectively. Determine (a) when the velocity is zero, (b) the velocity, the acceleration, and the total distance traveled when \( x = 0 \).

SOLUTION

We have \( x = t^3 - 6t^2 - 36t - 40 \)

Then \( v = \frac{dx}{dt} = 3t^2 - 12t - 36 \)

and \( a = \frac{dv}{dt} = 6t - 12 \)

(a) When \( v = 0 \):

\[
3t^2 - 12t - 36 = 3(t^2 - 4t - 12) = 0
\]

or \( (t + 2)(t - 6) = 0 \)

or \( t = -2 \text{ s (Reject)} \) and \( t = 6 \text{ s} \) \( t = 6.00 \text{ s} \)

(b) When \( x = 0 \):

\[
t^3 - 6t^2 - 36t - 40 = 0
\]

Factoring \( (t - 10)(t + 2)(t + 2) = 0 \) or \( t = 10 \text{ s} \)

Now observe that \( 0 \leq t < 6 \text{ s} : v < 0 \)

\( 6 \text{ s} < t \leq 10 \text{ s} : v > 0 \)

and at \( t = 0 \):

\( x_0 = -40 \text{ m} \)

\( t = 6 \text{ s} : x_6 = (6)^3 - 6(6)^2 - 36(6) - 40 \)

\( = -256 \text{ m} \)

\( t = 10 \text{ s} : v_{10} = 3(10)^2 - 12(10) - 36 \) or \( v_{10} = 144.0 \text{ m/s} \)

\( a_{10} = 6(10) - 12 \) or \( a_{10} = 48.0 \text{ m/s}^2 \)

Then

\[
|x_6 - x_0| = |-256 - (-40)| = 216 \text{ m}
\]

\( x_{10} - x_6 = 0 - (-256) = 256 \text{ m} \)

Total distance traveled = \((216 + 256) \text{ m} = 472 \text{ m} \)
PROBLEM 11.8

The motion of a particle is defined by the relation \( x = t^3 - 9t^2 + 24t - 8 \), where \( x \) and \( t \) are expressed in meters and seconds, respectively. Determine (a) when the velocity is zero, (b) the position and the total distance traveled when the acceleration is zero.

SOLUTION

We have \( x = t^3 - 9t^2 + 24t - 8 \)

Then \( v = \frac{dx}{dt} = 3t^2 - 18t + 24 \)

and \( a = \frac{dv}{dt} = 6t - 18 \)

(a) When \( v = 0 \):
\[ 3t^2 - 18t + 24 = 3(t^2 - 6t + 8) = 0 \]
\[ (t - 2)(t - 4) = 0 \]
\[ t = 2.00 \text{ s} \quad \text{and} \quad t = 4.00 \text{ s} \]

(b) When \( a = 0 \):
\[ 6t - 18 = 0 \quad \text{or} \quad t = 3 \text{ s} \]

At \( t = 3 \text{ s} \):
\[ x_3 = (3)^3 - 9(3)^2 + 24(3) - 8 \]
\[ \quad \text{or} \quad x_3 = 10.00 \text{ m} \]

First observe that \( 0 \leq t < 2 \text{ s} \):
\[ v > 0 \]
\[ 2 \text{ s} < t \leq 3 \text{ s} \]
\[ v < 0 \]

Now

At \( t = 0 \):
\[ x_0 = -8 \text{ m} \]

At \( t = 2 \text{ s} \):
\[ x_2 = (2)^3 - 9(2)^2 + 24(2) - 8 = 12 \text{ m} \]

Then
\[ x_2 - x_0 = 12 - (-8) = 20 \text{ m} \]
\[ |x_3 - x_2| = |10 - 12| = 2 \text{ m} \]

Total distance traveled = \((20 + 2) \text{ m} = 22.0 \text{ m} \)
PROBLEM 11.9

The acceleration of a particle is defined by the relation \( a = -8 \text{ m/s}^2 \). Knowing that \( x = 20 \text{ m} \) when \( t = 4 \text{ s} \) and that \( x = 4 \text{ m} \) when \( v = 16 \text{ m/s} \), determine (a) the time when the velocity is zero, (b) the velocity and the total distance traveled when \( t = 11 \text{ s} \).

SOLUTION

We have

\[
\frac{dv}{dt} = a = -8 \text{ m/s}^2
\]

Then

\[
\int dv = \int (-8 \, dt) + C \quad C = \text{constant}
\]

or

\[ v = -8t + C \text{ (m/s)} \]

Also

\[
\frac{dx}{dt} = v = -8t + C
\]

At \( t = 4 \text{ s}, x = 20 \text{ m}: \)

\[
\int_{20}^{x} dx = \int_{4}^{t} (-8t + C) \, dt
\]

or

\[ x - 20 = [-4t^2 + Ct + C]_{4}^{t} \]

or

\[ x = -4t^2 + C(t - 4) + 84 \text{ (m)} \]

When \( v = 16 \text{ m/s}, x = 4 \text{ m}: \)

\[ 16 = -8t + C \quad \Rightarrow \quad C = 16 + 8t \]

\[ 4 = -4t^2 + C(t - 4) + 84 \]

Combining

\[ 0 = -4t^2 + (16 + 8t)(t - 4) + 80 \]

Simplifying

\[ t^2 - 4t + 4 = 0 \]

or

\[ t = 2 \text{ s} \]

and

\[ C = 32 \text{ m/s} \]

\[ v = -8t + 32 \text{ (m/s)} \]

\[ x = -4t^2 + 32t - 44 \text{ (m)} \]

(a) \( \text{When } v = 0: \)

\[ -8t + 32 = 0 \quad \Rightarrow \quad t = 4 \text{ s} \]

(b) \( \text{Velocity and distance at 11 s.} \)

\[
v_{11} = -(8)(11) + 32 \quad \Rightarrow \quad v_{11} = -56.0 \text{ m/s} \]

At \( t = 0: \)

\[ x_{0} = 44 \text{ m} \]

at \( t = 4 \text{ s:} \)

\[ x_{4} = 20 \text{ m} \]

at \( t = 11 \text{ s:} \)

\[ x_{11} = -4(11)^2 + 32(11) - 44 = -176 \text{ m} \]
PROBLEM 11.9 (Continued)

Now observe that

\[0 \leq t < 4 \quad v > 0\]
\[4 \leq t \leq 11 \quad v < 0\]

Then

\[x_4 - x_0 = 20 - (-44) = 64 \text{ m}\]
\[|x_{11} - x_4| = |-176 - 20| = 196 \text{ m}\]

Total distance traveled = (64 + 196) m = 260 m
PROBLEM 11.10

The acceleration of a particle is directly proportional to the square of the time \( t \). When \( t = 0 \), the particle is at \( x = 24 \text{ m} \). Knowing that at \( t = 6 \text{ s} \), \( x = 96 \text{ m} \) and \( v = 18 \text{ m/s} \), express \( x \) and \( v \) in terms of \( t \).

SOLUTION

We have

\[ a = \text{constant} \]

Now

\[ \frac{dv}{dt} = a = kt^2 \]

At \( t = 6 \text{ s}, v = 18 \text{ m/s} \):

\[ \int_{18}^{v} dv = \int_{6}^{t} kt^2 dt \]

or

\[ v - 18 = \frac{1}{3} k(t^3 - 216) \]

or

\[ v = 18 + \frac{1}{3} k(t^3 - 216) \text{ (m/s)} \]

Also

\[ \frac{dx}{dt} = v = 18 + \frac{1}{3} k(t^3 - 216) \]

At \( t = 0, x = 24 \text{ m} \):

\[ \int_{24}^{x} dx = \int_{0}^{t} \left[ 18 + \frac{1}{3} k(t^3 - 216) \right] dt \]

or

\[ x - 24 = 18t + \frac{1}{3} k \left( \frac{1}{4} t^4 - 216t \right) \]

Now

At \( t = 6 \text{ s}, x = 96 \text{ m} \):

\[ 96 - 24 = 18(6) + \frac{1}{3} k \left( \frac{1}{4} (6)^4 - 216(6) \right) \]

or

\[ k = \frac{1}{9} \text{ m/s}^4 \]

Then

\[ x - 24 = 18t + \frac{1}{3} \left( \frac{1}{9} \right) \left( \frac{1}{4} t^4 - 216t \right) \]

or

\[ x(t) = \frac{1}{108} t^4 + 10t + 24 \]

and

\[ v = 18 + \frac{1}{3} \left( \frac{1}{9} \right) (t^3 - 216) \]

or

\[ v(t) = \frac{1}{27} t^3 + 10 \]
**PROBLEM 11.11**

The acceleration of a particle is directly proportional to the time \( t \). At \( t = 0 \), the velocity of the particle is \( v = 400 \) mm/s. Knowing that \( v = 375 \) mm/s and that \( x = 500 \) mm when \( t = 1 \) s, determine the velocity, the position, and the total distance traveled when \( t = 7 \) s.

**SOLUTION**

We have 

\[ a = kt \quad k = \text{constant} \]

Now 

\[ \frac{dv}{dt} = a = kt \]

At \( t = 0 \), \( v = 400 \) mm/s:

\[ \int_0^v dv = \int_0^k dt \]

or 

\[ v - 400 = \frac{1}{2}kt^2 \]

or 

\[ v = 400 + \frac{1}{2}kt^2 \text{ (mm/s)} \]

At \( t = 1 \) s, \( v = 375 \) mm/s:

\[ 375 \text{ mm/s} = 400 \text{ mm/s} + \frac{1}{2}k(1)^2 \]

or 

\[ k = -50 \text{ mm/s}^3 \quad \text{and} \quad v = 400 - 25t^2 \]

Also 

\[ \frac{dx}{dt} = v = 400 - 25t^2 = 25(16 - t^2) \]

At \( t = 1 \) s, \( x = 500 \) mm:

\[ \int_{500}^{x} dx = \int_{1}^{t} 25(16 - t^2) dt \]

or 

\[ x - 500 = 25 \left[ 16t - \frac{1}{3}t^3 \right]_1 \]

or 

\[ x = 25 \left[ -\frac{1}{3}t^3 + 16t \right] + 500 - 400 + \frac{25}{3} \]

Then 

\[ x = 25(-t^3/3 + 16t) + \frac{325}{3} \]

At \( t = 7 \) s:

\[ v_7 = 25\left\{ 16 - (7)^2 \right\} \quad \text{or} \quad v_7 = -825 \text{ mm/s} \]

\[ x_7 = 25\left\{ -\frac{1}{3}(7)^3 + 16(7) + \frac{13}{3} \right\} \quad \text{or} \quad x_7 = 50.0 \text{ mm} \]

When \( v = 0 \):

\[ 25(16-t^2) = 0 \quad \text{or} \quad t = 4 \text{ s} \]
PROBLEM 11.11 (Continued)

At \( t = 0 \):

\[ x_0 = \frac{325}{3} \]

\( t = 4 \text{ s} \):

\[ x_4 = 25 \left( -\frac{1}{3} \cdot 4^3 + 16 \cdot 4 + \frac{13}{3} \right) = 1175 \text{ mm} \]

Now observe that

\[ 0 \leq t < 4 \text{ s} \quad v > 0 \]

\[ 4 \text{ s} < t \leq 7 \text{ s} \quad v < 0 \]

Then

\[ x_4 - x_0 = 25 \left( 47 - \frac{13}{3} \right) = 1067 \text{ mm} \]

\[ |x_7 - x_4| = |50 - 1775| = 1125 \text{ mm} \]

Total distance traveled = \((1067 + 1125)\text{ mm} = 2192 \text{ mm}\)

2190 mm ▶
**PROBLEM 11.12**

The acceleration of a particle is defined by the relation $a = kt^2$. (a) Knowing that $v = -10 \text{ m/s}$ when $t = 0$ and that $v = +10 \text{ m/s}$ when $t = 4 \text{ s}$, determine the constant $k$. (b) Write the equations of motion, knowing also that $x = 0$ when $t = 4 \text{ s}$.

**SOLUTION**

$$a = kt^2$$

$$\frac{dv}{dt} = a = kt^2$$

$t = 0$, $v = -10 \text{ m/s}$ and $t = 4 \text{ s}$, $v = +10 \text{ m/s}$

(a)

$$\int_{-10}^{10} dv = \int_{0}^{4} kt^2 dt$$

$$10 - (-10) = \frac{1}{3}k(4)^3$$

$$k = 0.9375 \text{ m/s}^4$$

(b) Substituting $k = 0.9375 \text{ m/s}^4$ into (1)

$$\frac{dv}{dt} = a = 0.9375t^2$$

$t = 0$, $v = -10 \text{ m/s}$:

$$\int_{-10}^{v} dv = \int_{0}^{t} 0.9375t^2 dt$$

$$v - (-10) = \frac{1}{3}0.9375(t)^3$$

$$v = (0.3125t^3 - 10)$$

$$\frac{dx}{dt} = v = 0.3125t^3 - 10$$

$t = 4 \text{ s}$, $x = 0$:

$$\int_{0}^{x} dx = \int_{4}^{4} (0.3125t^3 - 10) dt; \quad x = \left[ \frac{0.3125t^4}{4} - 10t \right]_{4}^{4}$$

$$x = \left[ \frac{0.3125(4)^4}{4} - 10(4) \right] - \left[ \frac{0.3125(4)^4}{4} - 10(4) \right]$$

$$x = \left( \frac{0.3125(4)^4}{4} - 10(4) \right) - 19.968 + 40$$

$$x = 0.078125t^4 - 10t + 20.032$$
**PROBLEM 11.13**

The acceleration of a particle is defined by the relation $a = A - 6t^2$, where $A$ is a constant. At $t = 0$, the particle starts at $x = 8$ m with $v = 0$. Knowing that at $t = 1$ s, $v = 30$ m/s, determine (a) the times at which the velocity is zero, (b) the total distance traveled by the particle when $t = 5$ s.

**SOLUTION**

We have

$$a = A - 6t^2 \quad A = \text{constant}$$

Now

$$\frac{dv}{dt} = a = A - 6t^2$$

At $t = 0, v = 0$:

$$\int_0^v dv = \int_0^t (A - 6t^2)dt$$

or

$$v = At - 2t^3 (\text{m/s})$$

At $t = 1$ s, $v = 30$ m/s:

$$30 = A(1) - 2(1)^3$$

or

$$A = 32 \text{ m/s}^2 \quad \text{and} \quad v = 32t - 2t^3$$

Also

$$\frac{dx}{dt} = v = 32t - 2t^3$$

At $t = 0, x = 8$ m:

$$\int_8^x dx = \int_0^t (32t - 2t^3)dt$$

or

$$x = 8 + 16t^2 - \frac{1}{2}t^4 \quad (\text{m})$$

(a) When $v = 0$:

$$32t - 2t^3 = 2t(16 - t^2) = 0$$

or

$$t = 0 \quad \text{and} \quad t = 4.00 \text{ s}$$

(b) At $t = 4$ s:

$$x_4 = 8 + 16(4)^2 - \frac{1}{2}(4)^4 = 136 \text{ m}$$

At $t = 5$ s:

$$x_5 = 8 + 16(5)^2 - \frac{1}{2}(5)^4 = 95.5 \text{ m}$$
PROBLEM 11.13 (Continued)

Now observe that

\[ 0 < t < 4 \text{ s} \quad v > 0 \]
\[ 4 \text{ s} < t \leq 5 \text{ s} \quad v < 0 \]

Then

\[ x_4 - x_0 = 136 - 8 = 128 \text{ m} \]
\[ |x_5 - x_4| = |95.5 - 136| = 40.5 \text{ m} \]

Total distance traveled = \((128 + 40.5) \text{ m} = 168.5 \text{ m}\)
PROBLEM 11.14

It is known that from \( t = 2 \) s to \( t = 10 \) s the acceleration of a particle is inversely proportional to the cube of the time \( t \). When \( t = 2 \) s, \( v = -15 \) m/s, and when \( t = 10 \) s, \( v = 0.36 \) m/s. Knowing that the particle is twice as far from the origin when \( t = 2 \) s as it is when \( t = 10 \) s, determine (a) the position of the particle when \( t = 2 \) s, and when \( t = 10 \) s, (b) the total distance traveled by the particle from \( t = 2 \) s to \( t = 10 \) s.

SOLUTION

We have

\[ a = \frac{k}{t^3} \quad k = \text{constant} \]

Now

\[ \frac{dv}{dt} = a = \frac{k}{t^3} \]

At \( t = 2 \) s, \( v = -15 \) m/s:

\[ \int_{-15}^{v} dv = \int_{2}^{t} \frac{k}{t^3} dt \]

or

\[ v - (-15) = -\frac{k^2}{2} \left[ \frac{1}{t^2} - \frac{1}{(2)^2} \right] \]

or

\[ v = \frac{k}{2} \left( \frac{1}{4} - \frac{1}{t^2} \right) - 15 \text{ (m/s)} \]

At \( t = 10 \) s, \( v = 0.36 \) m/s:

\[ 0.36 = \frac{k}{2} \left( \frac{1}{4} - \frac{1}{10^2} \right) - 15 \]

or

\[ k = 128 \text{ m} \cdot \text{s} \]

and

\[ v = 1 - \frac{64}{t^2} \text{ (m/s)} \]

(a) We have

\[ \frac{dx}{dt} = v = 1 - \frac{64}{t^2} \]

Then

\[ \int dx = \int \left( 1 - \frac{64}{t^2} \right) dt + C \quad C = \text{constant} \]

or

\[ x = t + \frac{64}{t} + C \text{ (m)} \]

Now \( x_2 = 2x_{10} : 2 + \frac{64}{2} + C = 2 \left( 10 + \frac{64}{10} + C \right) \]

or

\[ C = 1.2 \text{ m} \]

and

\[ x = t + \frac{64}{t} + 1.2 \text{ (m)} \]
PROBLEM 11.14 (Continued)

At \( t = 2 \) s:
\[
x_2 = 2 + \frac{64}{2} + 1.2 \quad \text{or} \quad x_2 = 35.2 \text{ m} \quad \blacktriangle
\]

or

At \( t = 10 \) s:
\[
x_{10} = 10 + \frac{64}{10} + 1.2 \quad \text{or} \quad x_{10} = 17.60 \text{ m} \quad \blacktriangle
\]

Note: A second solution exists for the case \( x_2 > 0, x_{10} < 0 \). For this case, \( c = -22 \frac{4}{15} \text{ m} \)

and
\[
x_2 = 11 \frac{11}{15} \text{ m}, \quad x_{10} = -5 \frac{13}{15} \text{ m}
\]

(b) When \( v = 0 \):
\[
1 - \frac{64}{t^2} = 0 \quad \text{or} \quad t = 8 \text{ s}
\]

At \( t = 8 \) s:
\[
x_8 = 8 + \frac{64}{8} + 1.2 = 17.2 \text{ m}
\]

Now observe that \( 2 \text{ s} \leq t < 8 \text{ s} \): \( v < 0 \)

\( 8 \text{ s} < t \leq 10 \text{ s} \): \( v > 0 \)

Then
\[
|x_8 - x_2| = |17.2 - 35.2| = 18 \text{ m}
\]
\[
x_{10} - x_8 = 17.6 - 17.2 = 0.4 \text{ m}
\]

Total distance traveled = \((18 + 0.4) \text{ m} = 18.40 \text{ m} \quad \blacktriangle

Note: The total distance traveled is the same for both cases.
**PROBLEM 11.15**

The acceleration of a particle is defined by the relation \( a = -k/x \). It has been experimentally determined that \( v = 5 \) m/s when \( x = 0.2 \) m and that \( v = 3 \) m/s when \( x = 0.4 \) m. Determine (a) the velocity of the particle when \( x = 0.5 \) m, (b) the position of the particle at which its velocity is zero.

**SOLUTION**

\[
a = \frac{vdv}{dx} = -\frac{k}{x}
\]

Separate and integrate using \( x = 0.2 \), \( v = 5 \) m/s.

\[
\int_{5}^{v} vdv = -k \int_{0.2}^{x} \frac{dx}{x}
\]

\[
\frac{1}{2} v^{2} \bigg|_{0.2}^{v} = -k \ln x \bigg|_{0.2}^{x}
\]

\[
\frac{1}{2} v^{2} - \frac{1}{2} (5)^{2} = -k \ln \left( \frac{x}{0.2} \right) \tag{1}
\]

When \( v = 3 \) m/s, \( x = 0.4 \) m

\[
\frac{1}{2} (3)^{2} - \frac{1}{2} (5)^{2} = -k \ln \left( \frac{0.4}{0.2} \right)
\]

Solve for \( k \).

\( k = 11.5416 \) m²/s²

(a) Substitute \( k = 11.5416 \) m²/s² and \( x = 0.5 \) m into (1).

\[
\frac{1}{2} v^{2} - \frac{1}{2} (5)^{2} = -11.5416 \ln \left( \frac{0.5}{0.2} \right)
\]

\( v = 1.962 \) m/s

(b) For \( v = 0 \),

\[
0 - \frac{1}{2} (5)^{2} = -11.5416 \ln \left( \frac{x}{0.2} \right)
\]

\[
\ln \left( \frac{x}{0.2} \right) = 1.083
\]

\( x = 0.591 \) m
PROBLEM 11.16

A particle starting from rest at \( x = 0.3 \) m is accelerated so that its velocity doubles in magnitude between \( x = 0.6 \) m and \( x = 2.4 \) m. Knowing that the acceleration of the particle is defined by the relation \( a = k \left( x - \frac{A}{x} \right) \), determine the values of the constants \( A \) and \( k \) if the particle has a velocity of 8.7 m/s when \( x = 4.8 \) m.

SOLUTION

We have

\[
\frac{dv}{dx} = a = k \left( x - \frac{A}{x} \right)
\]

When \( x = 0.3 \), \( v = 0 \):

\[
\int_0^v dv = \int_{0.3}^x k \left( x - \frac{A}{x} \right) dx
\]

or

\[
\frac{1}{2} v^2 = k \left[ \frac{1}{2} x^2 - A \ln x \right]_{0.3}^x = k \left( \frac{1}{2} x^2 - 0.045 - A \ln \frac{x}{0.3} \right)
\]

At \( x = 0.6 \):

\[
\frac{1}{2} v^2_{0.6} = k \left[ \frac{1}{2} (0.6)^2 - A \ln \frac{0.6}{0.3} - 0.045 \right] = k(0.135 - A \ln 2)
\]

\( x = 2.4 \):

\[
\frac{1}{2} v^2_{2.4} = k \left[ \frac{1}{2} (2.4)^2 - A \ln \frac{2.4}{0.3} - 0.045 \right] = k(2.835 - A \ln 8)
\]

Now \( \frac{v_{2.4}}{v_{0.6}} = 2 \):

\[
\frac{\frac{1}{2} v^2_{2.4}}{\frac{1}{2} v^2_{0.6}} = (2)^2 = \frac{k(2.835 - A \ln 8)}{k(0.135 - A \ln 2)}
\]

or

\[
0.54 - 4 A \ln 2 = (2.835 - A \ln 8)
\]

or

\[
2.295 = A(\ln 8 - 4 \ln 2) = A(\ln 8 - \ln 2^4) = A \ln \left( \frac{1}{2} \right)
\]

or

\[
A = -3.31 \text{ m}^2
\]

When \( x = 4.8 \), \( v = 8.7 \) m/s:

\[
\frac{1}{2} (8.7)^2 = k \left[ \frac{1}{2} (4.8)^2 - \frac{2.295}{\ln(\frac{1}{2})} \ln \left( \frac{4.8}{0.3} \right) - 0.045 \right]
\]

Noting that

\[
\ln \left( \frac{4.8}{0.3} \right) = \ln(16) = 4 \ln 2 \quad \text{and} \quad \ln \left( \frac{1}{2} \right) = -\ln(2)
\]

We have

\[
37.845 = k[11.52 + 9.18 - 0.045]
\]

or

\[
k = 1.832 \text{ s}^{-2}
\]
PROBLEM 11.17

A particle oscillates between the points \( x = 40 \) mm and \( x = 160 \) mm with an acceleration \( a = k(100 - x) \), where \( a \) and \( x \) are expressed in mm/s\(^2\) and mm, respectively, and \( k \) is a constant. The velocity of the particle is 18 mm/s when \( x = 100 \) mm and is zero at both \( x = 40 \) mm and \( x = 160 \) mm. Determine (a) the value of \( k \), (b) the velocity when \( x = 120 \) mm.

SOLUTION

(a) We have 
\[
\frac{dv}{dx} = a = k(100 - x)
\]
When \( x = 40 \) mm, \( v = 0 \):
\[
\int_0^v dv = \int_{40}^x k(100 - x)dx
\]
or
\[
\frac{1}{2}v^2 = k\left[100x - \frac{1}{2}x^2\right]_{40}^x
\]
or
\[
\frac{1}{2}v^2 = k\left(100x - \frac{1}{2}x^2 - 3200\right)
\]
When \( x = 100 \) mm, \( v = 18 \) mm/s:
\[
\frac{1}{2}(18)^2 = k\left[100(100) - \frac{1}{2}(100)^2 - 3200\right]
\]
or
\[
k = 0.0900 \text{ s}^{-2}
\]
(b) When \( x = 120 \) mm:
\[
\frac{1}{2}v^2 = 0.09\left[100(120) - \frac{1}{2}(120)^2 - 3200\right] = 144
\]
or
\[
v = \pm 16.97 \text{ mm/s}
\]
PROBLEM 11.18

A particle starts from rest at the origin and is given an acceleration \( a = \frac{k}{(x+4)^2} \), where \( a \) and \( x \) are expressed in mm/s\(^2\) and m, respectively, and \( k \) is a constant. Knowing that the velocity of the particle is 4 m/s when \( x = 8 \) m, determine (a) the value of \( k \), (b) the position of the particle when \( v = 4.5 \) m/s, (c) the maximum velocity of the particle.

SOLUTION

(a) We have

\[
\frac{vdv}{dx} = a = \frac{k}{(x+4)^2}
\]

When \( x = 0, v = 0 \):

\[
\int_0^v dv = \int_0^x \frac{k}{(x+4)^2} \, dx
\]

or

\[
\frac{1}{2}v^2 = -k \left( \frac{1}{x+4} \right) - \frac{1}{4}
\]

When \( x = 8 \) m, \( v = 4 \) m/s:

\[
\frac{1}{2}(4)^2 = -k \left( \frac{1}{8+4} \right) - \frac{1}{4}
\]

or

\[
k = 48 \text{ m}^3/\text{s}^2
\]

(b) When \( v = 4.5 \) m/s:

\[
\frac{1}{2}(4.5)^2 = -48 \left( \frac{1}{x+4} \right) - \frac{1}{4}
\]

or

\[
x = 21.6 \text{ m}
\]

(c) Note that when \( v = v_{\text{max}} \), \( a = 0 \).

Now \( a \to 0 \) as \( x \to \infty \) so that

\[
\frac{1}{2}v_{\text{max}}^2 = 48 \ln \left( \frac{1}{4} - \frac{1}{x+4} \right) = 48 \left( \frac{1}{4} \right)
\]

or

\[
v_{\text{max}} = 4.90 \text{ m/s}
\]
**PROBLEM 11.19**

A piece of electronic equipment that is surrounded by packing material is dropped so that it hits the ground with a speed of 4 m/s. After impact the equipment experiences an acceleration of \( a = -kx \), where \( k \) is a constant and \( x \) is the compression of the packing material. If the packing material experiences a maximum compression of 20 mm, determine the maximum acceleration of the equipment.

**SOLUTION**

\[
a = \frac{vdv}{dx} = -kx
\]

Separate and integrate.

\[
\int_{v_0}^{v_f} v \, dv = -\int_0^{x_f} kx \, dx
\]

\[
\frac{1}{2}v_f^2 - \frac{1}{2}v_0^2 = -\frac{1}{2}kx_f^2
\]

Use \( v_0 = 4 \) m/s, \( x_f = 0.02 \) m, and \( v_f = 0 \). Solve for \( k \).

\[
0 - \frac{1}{2}(4)^2 = -\frac{1}{2}k(0.02)^2 \quad k = 40,000 \text{ s}^{-2}
\]

Maximum acceleration.

\[
a_{\text{max}} = -kx_{\text{max}} : \quad (-40,000)(0.02) = -800 \text{ m/s}^2
\]

\( a = 800 \text{ m/s}^2 \uparrow \)
PROBLEM 11.20

Based on experimental observations, the acceleration of a particle is defined by the relation $a = -(0.1 + \sin \frac{x}{b})$, where $a$ and $x$ are expressed in m/s$^2$ and meters, respectively. Knowing that $b = 0.8$ m and that $v = 1$ m/s when $x = 0$, determine (a) the velocity of the particle when $x = -1$ m, (b) the position where the velocity is maximum, (c) the maximum velocity.

SOLUTION

We have

$$v \frac{dv}{dx} = a = -(0.1 + \sin \frac{x}{0.8})$$

When $x = 0, v = 1$ m/s:

$$\int_{0}^{v} dv = \int_{0}^{x} \left(0.1 + \frac{-\sin x}{0.8}\right) dx$$

or

$$\frac{1}{2} (v^2 - 1) = \left[0.1x - \frac{0.8}{0.8} \cos \frac{x}{0.8}\right]_{0}^{x}$$

or

$$\frac{1}{2} v^2 = 0.1x + 0.8 \cos \frac{x}{0.8} - 0.3$$

(a) When $x = -1$ m:

$$\frac{1}{2} v^2 = 0.1(-1) + 0.8 \cos \frac{-1}{0.8} - 0.3$$

or

$$v = \pm 0.323 \text{ m/s}$$

(b) When $v = v_{\text{max}}, a = 0$:

$$-0.1 + \frac{0.8}{0.8} = 0$$

or

$$x = -0.080134 \text{ m} \quad x = -0.0801 \text{ m}$$

(c) When $x = -0.080134$ m:

$$\frac{1}{2} v_{\text{max}}^2 = -0.1(-0.080134) + 0.8 \cos \frac{-0.080134}{0.8} - 0.3$$

$$= 0.504 \text{ m}^2/\text{s}^2$$

or

$$v_{\text{max}} = 1.004 \text{ m/s}$$
PROBLEM 11.21

Starting from \( x = 0 \) with no initial velocity, a particle is given an acceleration \( a = 0.8\sqrt{v^2 + 49} \), where \( a \) and \( v \) are expressed in m/s\(^2\) and m/s, respectively. Determine (a) the position of the particle when \( v = 24 \) m/s, (b) the speed of the particle when \( x = 40 \) m.

SOLUTION

We have
\[
\frac{v}{\sqrt{x}} \cdot \frac{dv}{dx} = a = 0.8\sqrt{v^2 + 49}
\]

When \( x = 0, v = 0 \):
\[
\int_0^v \frac{vdv}{\sqrt{v^2 + 49}} = \int_0^x 0.8dx
\]

or
\[
\left[ \frac{v}{\sqrt{v^2 + 49}} \right]_0^v = 0.8x
\]

or
\[
\sqrt{v^2 + 49} - 7 = 0.8x
\]

(a) When \( v = 24 \) m/s:
\[
\sqrt{24^2 + 49} - 7 = 0.8x
\]

or
\[
x = 22.5 \text{ m}
\]

(b) When \( x = 40 \) m:
\[
\sqrt{v^2 + 49} - 7 = 0.8(40)
\]

or
\[
v = 38.4 \text{ m/s}
\]
PROBLEM 11.22

The acceleration of a particle is defined by the relation \( a = -k \sqrt{v} \), where \( k \) is a constant. Knowing that \( x = 0 \) and \( v = 81 \text{ m/s} \) at \( t = 0 \) and that \( v = 36 \text{ m/s} \) when \( x = 18 \text{ m} \), determine (a) the velocity of the particle when \( x = 20 \text{ m} \), (b) the time required for the particle to come to rest.

SOLUTION

(a) We have

\[
\frac{dv}{dx} = a = -k \sqrt{v}
\]

so that

\[
\sqrt{v} \, dv = -k \, dx
\]

When \( x = 0, \ v = 81 \text{ m/s} \):

\[
\int_{0}^{v} \sqrt{v} \, dv = \int_{0}^{x} -k \, dx
\]

or

\[
\frac{2}{3} (v^{3/2} - 279) = -k x
\]

or

\[
\frac{2}{3} (v^{3/2} - 729) = -k x
\]

When \( x = 18 \text{ m}, \ v = 36 \text{ m/s} \):

\[
\frac{2}{3} (36^{3/2} - 729) = -k (18)
\]

or

\[
k = 19 \text{ m/s}^2
\]

Finally

When \( x = 20 \text{ m} \):

\[
\frac{2}{3} (v^{3/2} - 729) = -19(20)
\]

or

\[
v^{3/2} = 159
\]

\( v = 29.3 \text{ m/s} \)

(b) We have

\[
\frac{dv}{dt} = a = -19 \sqrt{v}
\]

At \( t = 0, \ v = 81 \text{ m/s} \):

\[
\int_{0}^{v} \frac{dv}{\sqrt{v}} = \int_{0}^{t} -19 \, dt
\]

or

\[
2 \sqrt{v} \bigg|_{0}^{81} = -19 t
\]

or

\[
2(\sqrt{v} - 9) = -19 t
\]

When \( v = 0 \):

\[
2(-9) = -19 t
\]

or

\( t = 0.947 \text{ s} \)
PROBLEM 11.23

The acceleration of a particle is defined by the relation \( a = -0.8v \), where \( a \) is expressed in m/s\(^2\) and \( v \) in m/s. Knowing that at \( t = 0 \) the velocity is 1 m/s, determine (a) the distance the particle will travel before coming to rest, (b) the time required for the particle to come to rest, (c) the time required for the particle to be reduced by 50 percent of its initial value.

SOLUTION

(a) \[
a = \frac{vdv}{dx} = -0.8v \quad dv = -0.8dx
\]
Separate and integrate with \( v = 1 \) m/s when \( x = 0 \).
\[
\int_1^v dv = -0.8\int_0^x dx
\]
\[
v - 1 = -0.8x
\]
Distance traveled.
For \( v = 0 \),
\[
x = \frac{-1}{-0.8} \Rightarrow x = 1.25 \text{ m}
\]
(b) \[
a = \frac{dv}{dt} = -0.8v
\]
Separate.
\[
\int_1^v \frac{dv}{v} = -\int_0^x 0.8dt
\]
\[
\ln v - \ln 1 = -0.8t
\]
\[
\ln v = -0.8t \quad t = 1.25 \ln \left( \frac{1}{v} \right)
\]
For \( v = 0 \), we get \( t = \infty \).

(c) For \( v = 0.5 \) m/s,
\[
t = 1.25 \ln \left( \frac{1}{0.5} \right) = 0.866 \text{ s}
\]
**PROBLEM 11.24**

A bowling ball is dropped from a boat so that it strikes the surface of a lake with a speed of 8 m/s. Assuming the ball experiences a downward acceleration of $a = 3 - 0.1v^2$ when in the water, determine the velocity of the ball when it strikes the bottom of the lake.

**SOLUTION**

$v_0 = 8 \text{ m/s}, \quad x - x_0 = 10 \text{ m}$

$a = 3 - 0.1v^2 = k(c^2 - v^2)$

where

$k = 0.1 \text{ m}^{-1} \quad \text{and} \quad c^2 = \frac{3}{0.1} = 30 \text{ m}^2 \text{ s}^{-2}$

$c = 5.4772 \text{ m/s}$

Since $v_0 > c$, write

$$a = \frac{dv}{dx} = -k(v^2 - c^2)$$

$$\frac{vdv}{v^2 - c^2} = -kdx$$

Integrating,

$$\left. \frac{1}{2} \ln(v^2 - c^2) \right|_{v_0}^{v} = -k(x - x_0)$$

$$\ln \frac{v^2 - c^2}{v_0^2 - c^2} = -2k(x - x_0)$$

$$\frac{v^2 - c^2}{v_0^2 - c^2} = e^{-2k(x - x_0)}$$

$$v^2 = c^2 + (v_0^2 - c^2)e^{-2k(x - x_0)}$$

$$v^2 = 30 + [(8)^2 - 30]e^{-2(0.1)(10)}$$

$$v^2 = 30 + 601.4 e^{-20}$$

$$v = 5.88 \text{ m/s}$$
PROBLEM 11.25
The acceleration of a particle is defined by the relation \( a = 0.4(1 - kv) \), where \( k \) is a constant. Knowing that at \( t = 0 \) the particle starts from rest at \( x = 4 \) m and that when \( t = 15 \) s, \( v = 4 \) m/s, determine (a) the constant \( k \), (b) the position of the particle when \( v = 6 \) m/s, (c) the maximum velocity of the particle.

SOLUTION
(a) We have \( \frac{dv}{dt} = a = 0.4(1 - kv) \)

At \( t = 0, v = 0 \): \[ \int_{0}^{v} \frac{dv}{1 - kv} = \int_{0}^{15} 0.4 dt \]

or \[ \frac{1}{k} [\ln(1 - kv)]_{0}^{v} = 0.4t \]

or \[ \ln(1 - kv) = -0.4kt \]

At \( t = 15 \) s, \( v = 4 \) m/s: \[ \ln(1 - 4k) = -0.4k(15) \]

\[ = -6k \]

Solving yields \( k = 0.145703 \) s/m

or \[ k = 0.1457 \] s/m

(b) We have \( v \frac{dv}{dx} = a = 0.4(1 - kv) \)

When \( x = 4 \) m, \( v = 0 \): \[ \int_{0}^{v} \frac{v dv}{1 - kv} = \int_{4}^{x} 0.4 dx \]

Now \[ \frac{v}{1 - kv} = \frac{1}{k} + \frac{1}{1 - kv} \]

Then \[ \int_{0}^{v} \left[ \frac{1}{k} + \frac{1}{k(1 - kv)} \right] dv = \int_{4}^{x} 0.4 dx \]

or \[ \left[ \frac{-v}{k} + \frac{1}{k^2} \ln(1 - kv) \right]_{0}^{v} = 0.4[x]_{4}^{x} \]

or \[ -\left[ \frac{v}{k} + \frac{1}{k^2} \ln(1 - kv) \right]_{0}^{v} = 0.4(x - 4) \]

When \( v = 6 \) m/s: \[ -\left[ \frac{6}{0.145703} + \frac{1}{(0.145703)^2} \ln(1 - 0.145703 \times 6) \right] = 0.4(x - 4) \]

or \[ 0.4(x - 4) = 56.4778 \]

or \[ x = 45.2 \] m
(c) The maximum velocity occurs when \( a = 0 \).

\[
a = 0: \quad 0.4(1 - k v_{\text{max}}) = 0
\]

or

\[
0.145703
\]

or

\[
v_{\text{max}} = 6.86 \text{ m/s} \quad \blacktriangleleft
\]

An alternative solution is to begin with Eq. (1).

\[
\ln(1 - kv) = -0.4kt
\]

Then

\[
v = \frac{1}{k}(1 - k^{-0.4kt})
\]

Thus, \( v_{\text{max}} \) is attained as \( t \to \infty \)

\[
v_{\text{max}} = \frac{1}{k}
\]

as above.
PROBLEM 11.26

A particle is projected to the right from the position $x = 0$ with an initial velocity of 9 m/s. If the acceleration of the particle is defined by the relation $a = -0.6v^{2/3}$, where $a$ and $v$ are expressed in m/s$^2$ and m/s, respectively, determine (a) the distance the particle will have traveled when its velocity is 4 m/s, (b) the time when $v = 1$ m/s, (c) the time required for the particle to travel 6 m.

SOLUTION

(a) We have $$\frac{dv}{dx} = a = -0.6v^{2/3}$$

When $x = 0, v = 9$ m/s: $$\int_{9}^{v} \frac{v^{2/3}}{v} dv = \int_{0}^{x} -0.6 dx$$

or $$-2v^{1/2} |_{0}^{9} = -0.6x$$

or $$x = \frac{1}{0.3} (3 - v^{1/2})$$

When $v = 4$ m/s:

or $$x = \frac{1}{0.3} (3 - 4^{1/2})$$

or $$x = 3.33 \text{ m } \triangleright$$

(b) We have $$\frac{dv}{dt} = a = -0.6v^{2/3}$$

When $t = 0, v = 9$ m/s: $$\int_{9}^{v} \frac{v^{2/3}}{v} dv = \int_{0}^{t} -0.6 dt$$

or $$-2v^{1/2} |_{0}^{9} = -0.6t$$

or $$\frac{1}{\sqrt{v}} - \frac{1}{3} = 0.3t$$

When $v = 1$ m/s:

or $$\frac{1}{\sqrt{1}} - \frac{1}{3} = 0.3t$$

or $$t = 2.22 \text{ s } \triangleright$$

(c) We have $$\frac{1}{\sqrt{v}} - \frac{1}{3} = 0.3t$$

or $$v = \left( \frac{3}{1 + 0.9t} \right)^{2} = \frac{9}{(1 + 0.9t)^{2}}$$

Now $$\frac{dx}{dt} = v = \frac{9}{(1 + 0.9t)^{2}}$$
PROBLEM 11.26 (Continued)

At \( t = 0, \ x = 0 \):

\[
\int_0^x dx = \int_0^t \frac{9}{(1+0.9t)^2} dt
\]

or

\[
x = 9 \left[ -\frac{1}{0.9} \frac{1}{1+0.9t} \right]_0^t
\]

= \( 10 \left( 1 - \frac{1}{1+0.9t} \right) \)

= \( \frac{9t}{1+0.9t} \)

When \( x = 6 \ m \):

\[
6 = \frac{9t}{1+0.9t}
\]

or

\[
t = 1.667 \ s \quad \blacktriangledown
\]

An alternative solution is to begin with Eq. (1).

\[
x = \frac{1}{0.3} (3 - v^{1/2})
\]

Then

\[
\frac{dx}{dt} = v = (3 - 0.3x)^2
\]

Now

At \( t = 0, \ x = 0 \):

\[
\int_0^x \frac{dx}{(3-0.3x)^2} = \int_0^t \frac{dt}{(3-0.3x)^2}
\]

or

\[
t = \frac{1}{0.3} \left[ \frac{1}{3-0.3x} \right]_0^x = \frac{x}{9-0.9x}
\]

Which leads to the same equation as above.
PROBLEM 11.27

Based on observations, the speed of a jogger can be approximated by the relation \( v = 12(1 - 0.06x)^{0.3} \), where \( v \) and \( x \) are expressed in km/h and km, respectively. Knowing that \( x = 0 \) at \( t = 0 \), determine (a) the distance the jogger has run when \( t = 1 \) h, (b) the jogger’s acceleration in m/s\(^2\) at \( t = 0 \), (c) the time required for the jogger to run 9 km.

SOLUTION

(a) We have

\[
\frac{dx}{dt} = v = 12(1 - 0.06x)^{0.3}
\]

At \( t = 0 \), \( x = 0 \):

\[
\int_0^x \frac{dx}{(1 - 0.06x)^{0.3}} = \int_0^t 12 dt
\]

or

\[
\frac{1}{0.7} \left( - \frac{1}{0.06} \right) [(1 - 0.06x)^{0.7}]_0^x = 12t
\]

or

\[
1 - (1 - 0.06x)^{0.7} = 0.504 t
\]

or

\[
x = \frac{1}{0.06} \left[ 1 - (1 - 0.504 t)^{0.7} \right]
\]

At \( t = 1 \) h

\[
x = \frac{1}{0.06} \left[ 1 - (1 - 0.504)(1)^{0.7} \right]
\]

or

\[
x = 10.55 \text{ km} \]

(b) We have

\[
a = \frac{dv}{dx}
\]

\[
= 12(1 - 0.06x)^{0.3} \frac{d}{dx} [12(1 - 0.06x)^{0.3}]
\]

\[
= 12^2(1 - 0.06x)^{0.3} (0.3)(-0.06)(1 - 0.06x)^{-0.7}
\]

\[
= -2.592(1 - 0.06x)^{-0.4}
\]

At \( t = 0 \), \( x = 0 \):

\[
a_0 = -\frac{2.592}{h^2} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \left( \frac{1 \text{ h}}{3600} \right)^2
\]

or

\[
a_0 = -2 \times 10^{-4} \text{ m/s}^2
\]

(c) From Eq. (1)

\[
t = \frac{1}{0.504} \left[ 1 - (1 - 0.06x)^{0.7} \right]
\]

When \( x = 9 \) km:

\[
t = \frac{1}{0.504} \left[ 1 - (1 - 0.06)(9)^{0.7} \right]
\]

\[
t = 0.832 \text{ hrs}
\]

or

\[
t = 49.9 \text{ min}
\]
PROBLEM 11.28

Experimental data indicate that in a region downstream of a given louvered supply vent, the velocity of the emitted air is defined by $v = \frac{0.18v_0}{x}$, where $v$ and $x$ are expressed in m/s and meters, respectively, and $v_0$ is the initial discharge velocity of the air. For $v_0 = 3.6$ m/s, determine (a) the acceleration of the air at $x = 2$ m, (b) the time required for the air to flow from $x = 1$ to $x = 3$ m.

SOLUTION

(a) We have

$$a = v \frac{dv}{dx} = \frac{0.18v_0}{x} \frac{d}{dx} \left( \frac{0.18v_0}{x} \right) = -\frac{0.0324v_0^2}{x^3}$$

When $x = 2$ m:

$$a = -\frac{0.0324(3.6)^2}{(2)^3}$$

or

$$a = -0.0525 \text{ m/s}^2$$

(b) We have

$$\frac{dx}{dt} = v = \frac{0.18v_0}{x}$$

From $x = 1$ m to $x = 3$ m:

$$\int_1^3 x \, dx = \int_{t_1}^{t_3} 0.18v_0 \, dt$$

or

$$\left[ \frac{1}{2} x^2 \right]_1^3 = 0.18v_0 (t_3 - t_1)$$

or

$$\left( \frac{1}{2} \cdot 3^2 \right) = 0.18v_0 (t_3 - t_1)$$

or

$$t_3 - t_1 = \frac{\frac{1}{2}(9 - 1)}{0.18(3.6)}$$

or

$$t_3 - t_1 = 6.17 \text{ s}$$
PROBLEM 11.29

The acceleration due to gravity at an altitude $y$ above the surface of the earth can be expressed as

$$a = \frac{-9.81}{[1 + (y/6.37 \times 10^6)]^2}$$

where $a$ and $y$ are expressed in m/s$^2$ and m, respectively. Using this expression, compute the height reached by a projectile fired vertically upward from the surface of the earth if its initial velocity is (a) 540 m/s, (b) 900 m/s, (c) 11,180 m/s.

SOLUTION

We have

$$\frac{dv}{dy} = a = \frac{-9.81}{[1 + \frac{y}{6.37 \times 10^6}]^2}$$

When

$$y = 0, \quad v = v_0$$

$$y = y_{\text{max}}, \quad v = 0$$

Then

$$\int_{v_0}^{0} v \, dv = \int_{0}^{y_{\text{max}}} \frac{-9.81}{[1 + \frac{y}{6.37 \times 10^6}]^2} \, dy$$

or

$$-\frac{1}{2} v_0^2 = -9.81 \left[ \frac{1}{6.37 \times 10^6} \frac{1}{1 + \frac{y}{6.37 \times 10^6}} \right]_0^{y_{\text{max}}}$$

or

$$v_0^2 = 124.9794 \times 10^6 \left[ 1 - \frac{1}{1 + \frac{1}{6.37 \times 10^6} y_{\text{max}}} \right]$$

or

$$y_{\text{max}} = 6.37 \times 10^6 \left[ \frac{v_0^2}{124.9794 \times 10^6 - v_0^2} \right]$$

(a) $v_0 = 540$ m/s:

$$y_{\text{max}} = 6.37 \times 10^6 \left[ \frac{540^2}{124.9794 \times 10^6 - 540^2} \right]$$

or

$$y_{\text{max}} = 14.90 \times 10^3 \text{ m}$$

(b) $v_0 = 900$ m/s:

$$y_{\text{max}} = 6.37 \times 10^6 \left[ \frac{900^2}{124.9794 \times 10^6 - 900^2} \right]$$

or

$$y_{\text{max}} = 41.6 \times 10^3 \text{ m}$$
PROBLEM 11.29 (Continued)

(c) $v_0 = 11,180 \text{ m/s}$:

$$y_{\text{max}} = 6.37 \times 10^6 \left[ \frac{(11,180)^2}{(124.9794 \times 10^6 - 11,180^2)} \right]$$

or

$$y_{\text{max}} = -6.12 \times 10^{10} \text{ m}$$

The velocity 11,180 m/s is approximately the escape velocity $v_R$ from the earth. For $v_R$

$$y_{\text{max}} \rightarrow \infty$$
PROBLEM 11.30

The acceleration due to gravity of a particle falling toward the earth is \( a = -\frac{gR^2}{r^2} \), where \( r \) is the distance from the center of the earth to the particle, \( R \) is the radius of the earth, and \( g \) is the acceleration due to gravity at the surface of the earth. If \( R = 6370 \) km calculate the escape velocity, that is, the minimum velocity with which a particle must be projected vertically upward from the surface of the earth if it is not to return to the earth. (Hint: \( v = 0 \) for \( r = \infty \)).

SOLUTION

We have

\[ \frac{dv}{dr} = a = -\frac{gR^2}{r^2} \]

When

\( r = R, \quad v = v_e \)
\( r = \infty, \quad v = 0 \)

Then

\[ \int_{v_e}^{0} v dv = \int_{R}^{\infty} -\frac{gR^2}{r^2} dr \]

or

\[ -\frac{1}{2}v_e^2 = gR^2 \left[ \frac{1}{r} \right]_R^{\infty} \]

or

\[ v_e = \sqrt{2gR} \]

or

\[ v_e = \left( 2 \times 9.81 \text{ m/s}^2 \times 6370 \text{ km} \times \frac{1000 \text{ m}}{1 \text{ km}} \right)^{1/2} \]

or

\[ v_e = 11,180 \text{ m/s} \]
PROBLEM 11.31

The velocity of a particle is \( v = v_0 [1 - \sin(\pi t/T)] \). Knowing that the particle starts from the origin with an initial velocity \( v_0 \), determine (a) its position and its acceleration at \( t = 3T \), (b) its average velocity during the interval \( t = 0 \) to \( t = T \).

SOLUTION

(a) We have

\[
\frac{dx}{dt} = v = v_0 \left[ 1 - \sin \left( \frac{\pi t}{T} \right) \right]
\]

At \( t = 0, x = 0 \):

\[
\int_0^x dx = \int_0^t v_0 \left[ 1 - \sin \left( \frac{\pi t}{T} \right) \right] dt
\]

or

\[
x = v_0 \left[ t + \frac{T}{\pi} \cos \left( \frac{\pi t}{T} \right) \right]_0
\]

\[
= v_0 \left[ t + \frac{T}{\pi} \cos \left( \frac{\pi t}{T} \right) - \frac{T}{\pi} \right]
\]

(1)

At \( t = 3T \):

\[
x_{3T} = v_0 \left[ 3T + \frac{T}{\pi} \cos \left( \frac{\pi \times 3T}{T} \right) - \frac{T}{\pi} \right]
\]

or

\[
x_{3T} = 2.36 v_0 T
\]

Also

\[
a = \frac{dv}{dt} = \frac{d}{dt} \left[ v_0 \left[ 1 - \sin \left( \frac{\pi t}{T} \right) \right] \right] = -v_0 \frac{\pi}{T} \cos \frac{\pi t}{T}
\]

At \( t = 3T \):

\[
a_{3T} = -v_0 \frac{\pi}{T} \cos \frac{\pi \times 3T}{T}
\]

or

\[
a_{3T} = \frac{\pi v_0}{T}
\]

(b) Using Eq. (1)

At \( t = 0 \):

\[
x_0 = v_0 \left[ 0 + \frac{T}{\pi} \cos(0) - \frac{T}{\pi} \right] = 0
\]
PROBLEM 11.31 (Continued)

At \( t = T \):  

\[
x_T = v_0 \left[ T + \frac{T}{\pi} \cos \left( \frac{\pi T}{T} \right) - \frac{T}{\pi} \right] 
\]

\[
= v_0 \left( T - \frac{2T}{\pi} \right) 
\]

\[
= 0.363v_0T 
\]

Now  

\[
v_{\text{ave}} = \frac{x_T - x_0}{\Delta t} = \frac{0.363v_0T - 0}{T - 0} 
\]

or  

\[
v_{\text{ave}} = 0.363v_0 \uparrow
\]
**PROBLEM 11.32**

The velocity of a slider is defined by the relation \( v = \sqrt{v^2 \sin(\omega_n t + \phi)} \). Denoting the velocity and the position of the slider at \( t = 0 \) by \( v_0 \) and \( x_0 \), respectively, and knowing that the maximum displacement of the slider is \( 2x_0 \), show that (a) \( v = \left( v_0^2 + x_0^2 \omega_n^2 \right) / 2x_0 \omega_n \), (b) the maximum value of the velocity occurs when \( x = x_0 \left[ 3 - (v_0 / x_0 \omega_n)^2 \right] / 2 \).

**SOLUTION**

(a) \( \text{At } t = 0, v = v_0: \quad v_0 = \sqrt{v^2 \sin(0 + \phi)} = \sqrt{v^2} \sin \phi \)

Then \( \cos \phi = \sqrt{v^2 - v_0^2} / v \) \( \sqrt{v^2} \sin(\omega_n t + \phi) \)

Now \( \frac{dx}{dt} = v = \sqrt{v^2 \sin(\omega_n t + \phi)} \)

At \( t = 0, x = x_0: \quad \int_{x_0}^{x} dx = \int_{0}^{t} \sqrt{v^2 \sin(\omega_n t + \phi)} dt \)

or \( x - x_0 = \sqrt{-\frac{1}{\omega_n} \cos(\omega_n t + \phi)} \)

or \( x = x_0 + \frac{\sqrt{v^2}}{\omega_n} \left[ \cos \phi - \cos(\omega_n t + \phi) \right] \)

Now observe that \( x_{\text{max}} \) occurs when \( \cos(\omega_n t + \phi) = -1 \). Then

\( x_{\text{max}} = 2x_0 = x_0 + \frac{\sqrt{v^2}}{\omega_n} \left[ \cos \phi - (-1) \right] \)

Substituting for \( \cos \phi \)

\( x_0 = \frac{\sqrt{v^2 - v_0^2}}{\sqrt{v^2} + 1} \)

or \( x_0 \omega_n - \sqrt{v^2 - v_0^2} \)

Squaring both sides of this equation

\( x_0^2 \omega_n^2 - 2x_0 \omega_n + v^2 = v^2 - v_0^2 \)

or \( v = \frac{v_0^2 + x_0^2 \omega_n^2}{2x_0 \omega_n} \)

Q. E. D.
PROBLEM 11.32 (Continued)

(b) First observe that $v_{\text{max}}$ occurs when $\omega_n t + \phi = \frac{\pi}{2}$. The corresponding value of $x$ is

$$x_{\text{max}} = x_0 + \frac{\sqrt{v^2 - v_0^2}}{\omega_n}$$

$$= x_0 + \frac{\sqrt{v^2 - v_0^2}}{\omega_n} \cos \phi$$

Substituting first for $\cos \phi$ and then for $v'$

$$x_{\text{max}} = x_0 + \frac{\sqrt{v^2 - v_0^2}}{\omega_n}$$

$$= x_0 + \frac{1}{\omega_n \omega_o} \left[ \left( \frac{v_0^2}{2} + \frac{x_0^2 \omega_o^2}{2} - v_0^2 \right) - v_0^2 \right]$$

$$= x_0 + \frac{1}{2 \omega_o \omega_n^2} \left( v_0^2 + 2v_0^2 x_0^2 \omega_o^2 + x_0^2 \omega_o^2 - 4x_0^2 \omega_o^2 v_0^2 \right)^{1/2}$$

$$= x_0 + \frac{1}{2 \omega_o \omega_n^2} \left[ \left( \frac{x_0^2 \omega_o^2}{v_0^2} - v_0^2 \right) \right]^{1/2}$$

$$= x_0 + \frac{x_0^2 \omega_o^2 - v_0^2}{2 \omega_o \omega_n^2}$$

$$= \frac{x_0}{2} \left[ 3 - \left( \frac{v_0}{\omega_o \omega_n^2} \right)^2 \right]$$

Q. E. D.
**PROBLEM 11.33**

A motorist enters a freeway at 45 km/h and accelerates uniformly to 99 km/h. From the odometer in the car, the motorist knows that she traveled 0.2 km while accelerating. Determine (a) the acceleration of the car, (b) the time required to reach 99 km/h.

---

**SOLUTION**

(a) \textit{Acceleration of the car.}

\[
v_1^2 = v_0^2 + 2a(x_1 - x_0)
\]

\[
a = \frac{v_1^2 - v_0^2}{2(x_1 - x_0)}
\]

Data:

- \(v_0 = 45 \text{ km/h} = 12.5 \text{ m/s}\)
- \(v_1 = 99 \text{ km/h} = 27.5 \text{ m/s}\)
- \(x_0 = 0\)
- \(x_1 = 0.2 \text{ km} = 200 \text{ m}\)

\[
a = \frac{(27.5)^2 - (12.5)^2}{2(200 - 0)} = 1.500 \text{ m/s}^2
\]

(b) \textit{Time to reach 99 km/h.}

\[
v_1 = v_0 + a(t_1 - t_0)
\]

\[
t_1 - t_0 = \frac{v_1 - v_0}{a}
\]

\[
t_1 - t_0 = \frac{27.5 - 12.5}{1.500} = 10.00 \text{ s}
\]
PROBLEM 11.34

A truck travels 220 m in 10 s while being decelerated at a constant rate of 0.6 m/s². Determine (a) its initial velocity, (b) its final velocity, (c) the distance traveled during the first 1.5 s.

SOLUTION

(a) Initial velocity.
\[ x = x_0 + v_0 t + \frac{1}{2} at^2 \]
\[ v_0 = \frac{x - x_0}{t} - \frac{1}{2} at \]
\[ = \frac{220}{10} - \frac{1}{2} (-0.6)(10) \]
\[ v_0 = 25.9 \text{ m/s} \]

(b) Final velocity.
\[ v = v_0 + at \]
\[ v = 25.0 + (-0.6)(10) \]
\[ v_f = 19.00 \text{ m/s} \]

(c) Distance traveled during first 1.5 s.
\[ x = x_0 + v_0 t + \frac{1}{2} at^2 \]
\[ = 0 + (25.0)(1.5) + \frac{1}{2} (-0.6)(1.5)^2 \]
\[ x = 36.8 \text{ m} \]
**PROBLEM 11.35**

Assuming a uniform acceleration of 3.3 m/s² and knowing that the speed of a car as it passes A is 45 km/h, determine (a) the time required for the car to reach B, (b) the speed of the car as it passes B.

---

**SOLUTION**

(a) **Time required to reach B.**

\[ v_A = 45 \text{ km/h} = 12.5 \text{ m/s}, \quad x_A = 0, \quad x_B = 50 \text{ m}, \quad a = 3.3 \text{ m/s}^2 \]

\[ x_B = x_A + v_A t + \frac{1}{2} at^2 \]

\[ 50 = 0 + 12.5t + \frac{1}{2} (3.3)t^2 \]

\[ 3.3t^2 + 25t - 100 = 0 \]

\[ t = \frac{-25 \pm \sqrt{(25)^2 - 4(3.3)(-100)}}{2(3.3)} \]

\[ t = 2.8943 \text{ s} \quad \text{or} \quad -10.47 \text{ sec} \]

Rejecting the negative root. \[ t = 2.8943 \text{ s} \]

(b) **Speed at B.**

\[ v_B = v_A + at = 12.5 + (3.3)(2.8943) = 22.0512 \text{ m/s} \]

\[ v_B = 79.4 \text{ km/h} \]
**PROBLEM 11.36**

A group of students launches a model rocket in the vertical direction. Based on tracking data, they determine that the altitude of the rocket was 27 m at the end of the powered portion of the flight and that the rocket landed 16 s later. Knowing that the descent parachute failed to deploy so that the rocket fell freely to the ground after reaching its maximum altitude and assuming that \( g = 9.81 \text{ m/s}^2 \), determine (a) the speed \( v_1 \) of the rocket at the end of powered flight, (b) the maximum altitude reached by the rocket.

**SOLUTION**

(a) We have

\[
y = y_1 + v_1 t + \frac{1}{2} a t^2
\]

At \( t_{\text{land}} \),

\[
y = 0
\]

Then

\[
0 = 27 \text{ m} + v_1 (16 \text{ s})
\]

\[
+ \frac{1}{2} (-9.81 \text{ m/s}^2) (16 \text{ s})^2
\]

or

\[
v_1 = 76.7925 \text{ m/s}
\]

\[
v_1 = 76.8 \text{ m/s} \uparrow
\]

(b) We have

\[
v^2 = v_1^2 + 2a(y - y_1)
\]

At

\[
y = y_{\text{max}}\,\,\, \, v = 0
\]

Then

\[
0 = (76.7925 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2) (y_{\text{max}} - 27 \text{ m})
\]

or

\[
y_{\text{max}} = 327.57 \text{ m}
\]

\[
y_{\text{max}} = 328 \text{ m} \uparrow
\]
PROBLEM 11.37
A sprinter in a 100-m race accelerates uniformly for the first 35 m and then runs with constant velocity. If the sprinter's time for the first 35 m in 5.4 s, determine (a) his acceleration, (b) his final velocity, (c) his time for the race.

SOLUTION
Given:
0 ≤ x ≤ 35 m,  a = constant
35 m < x ≤ 100 m,  v = constant
At t = 0,  v = 0  when x = 35 m,  t = 5.4 s

Find:
(a)  a
(b)  v when x = 100 m
(c)  t when x = 100 m

(a)  We have
x = 0 + 0t + \frac{1}{2}at²  for  0 ≤ x ≤ 35 m
At t = 5.4 s:
35 m = \frac{1}{2}a(5.4 s)²
or
a = \frac{35}{\frac{1}{2}a(5.4 s)²}  \Rightarrow  a = 2.4005 m/s²

(b)  First note that v = v_max  for  35 m ≤ x ≤ 100 m.
Now
v² = 0 + 2a(x - 0)  for  0 ≤ x ≤ 35 m
When x = 35 m:
v_max² = 2(2.4005 m/s²)(35 m)
or
v_max = 12.9628 m/s

(c)  We have
x = x_f + v_0(t - t_1)  for  35 m < x ≤ 100 m
When x = 100 m:
100 m = 35 m + (12.9628 m/s)(t_2 - 5.4) s
or
t_2 = 10.41 s
PROBLEM 11.38

A small package is released from rest at A and moves along the skate wheel conveyor ABCD. The package has a uniform acceleration of 4.8 m/s² as it moves down sections AB and CD, and its velocity is constant between B and C. If the velocity of the package at D is 7.2 m/s, determine (a) the distance d between C and D, (b) the time required for the package to reach D.

SOLUTION

(a) For A → B
and C → D
we have

\[ v^2 = v_0^2 + 2a(x-x_0) \]

Then,

at B

\[ v_{BC}^2 = 0 + 2(4.8 \text{ m/s}^2)(3-0) \text{ m} \]
\[ = 28.8 \text{ m}^2/\text{s}^2 \]

\[ (v_{BC} = 5.3666 \text{ m/s}) \]

and at D

\[ v_D^2 = v_{BC}^2 + 2a_{CD}(x_D-x_C) \]
\[ d = x_D-x_C \]

or

\[ (7.2 \text{ m/s})^2 = (28.8 \text{ m}^2/\text{s}^2) + 2(4.8 \text{ m/s}^2)d \]

or

\[ d = 2.40 \text{ m} \]

(b) For A → B
and C → D,
we have

\[ v = v_0 + at \]

Then

A → B

\[ 5.3666 \text{ m/s} = 0 + (4.8 \text{ m/s}^2)t_{AB} \]

or

\[ t_{AB} = 1.11804 \text{ s} \]

and C → D

\[ 7.2 \text{ m/s} = 5.3666 \text{ m/s} + (4.8 \text{ m/s}^2)t_{CD} \]

or

\[ t_{CD} = 0.38196 \text{ s} \]
PROBLEM 11.38 (Continued)

Now, for \( B \rightarrow C \),

we have \( x_C = x_B + v_{BC} t_{BC} \)

or \( 3 \text{ m} = (5.3666 \text{ m/s}) t_{BC} \)

or \( t_{BC} = 0.55901 \text{ s} \)

Finally,

\[ t_D = t_{AB} + t_{BC} + t_{CD} = (1.11804 + 0.55901 + 0.38196) \text{ s} \]

or \[ t_D = 2.06 \text{ s} \]
PROBLEM 11.39

A police officer in a patrol car parked in a 70 km/h speed zone observes a passing automobile traveling at a slow, constant speed. Believing that the driver of the automobile might be intoxicated, the officer starts his car, accelerates uniformly to 90 km/h in 8 s, and, maintaining a constant velocity of 90 km/h, overtakes the motorist 42 s after the automobile passed him. Knowing that 18 s elapsed before the officer began pursuing the motorist, determine (a) the distance the officer traveled before overtaking the motorist, (b) the motorist's speed.

SOLUTION

\( (v_p)_{26} = 90 \text{ km/h} = 25 \text{ m/s} \quad (v_p)_{42} = 90 \text{ km/h} = 25 \text{ m/s} \)

(a) Patrol car:

For \(18 \text{ s} < t \leq 26 \text{ s} \):
\[ v_p = 0 + a_p (t - 18) \]

At \(t = 26 \text{ s} \):
\[ 25 \text{ m/s} = a_p (26 - 18) \text{ s} \]

or
\[ a_p = 3.125 \text{ m/s}^2 \]

Also,
\[ x_p = 0 + 0(t - 18) - \frac{1}{2} a_p (t - 18)^2 \]

At \(t = 26 \text{ s} \):
\[ (x_p)_{26} = \frac{1}{2} (3.125 \text{ m/s}^2) (26 - 18)^2 = 100 \text{ m} \]

For \(26 \text{ s} < t \leq 42 \text{ s} \):
\[ x_p = (x_p)_{26} + (v_p)_{26} (t - 26) \]

At \(t = 42 \text{ s} \):
\[ (x_p)_{42} = 100 \text{ m} + (25 \text{ m/s})(42 - 26) \text{ s} \]
\[ = 500 \text{ m} \]

\[ (x_p)_{42} = 0.5 \text{ km} \]

(b) For the motorist's car:
\[ x_M = 0 + v_M t \]

At \(t = 42 \text{ s} \), \(x_M = x_p\):
\[ 500 \text{ m} = v_M (42 \text{ s}) \]

or
\[ v_M = 11.9048 \text{ m/s} \]

or
\[ v_M = 42.9 \text{ km/h} \]
PROBLEM 11.40

As relay runner A enters the 20-m-long exchange zone with a speed of 12.9 m/s, he begins to slow down. He hands the baton to runner B 1.82 s later as they leave the exchange zone with the same velocity. Determine (a) the uniform acceleration of each of the runners, (b) when runner B should begin to run.

SOLUTION

(a) For runner A:

\[ x_A = x_0 + \left( v_A \right)_0 t + \frac{1}{2} a_A t^2 \]

At \( t = 1.82 \) s:

\[ 20 \text{ m} = (12.9 \text{ m/s})(1.82 \text{ s}) + \frac{1}{2} a_A (1.82 \text{ s})^2 \]

or

\[ a_A = -2.10 \text{ m/s}^2 \]

Also

\[ v_A = \left( v_A \right)_0 + a_A t \]

At \( t = 1.82 \) s:

\[ v_A (1.82) = (12.9 \text{ m/s}) + (-2.10 \text{ m/s}^2)(1.82 \text{ s}) \]

\[ = 9.078 \text{ m/s} \]

For runner B:

\[ v_B^2 = 0 + 2a_B [x_B - 0] \]

When

\[ x_B = 20 \text{ m}, \quad v_B = v_A: \quad (9.078 \text{ m/s})^2 + 2a_B (20 \text{ m}) \]

or

\[ a_B = 2.0603 \text{ m/s}^2 \]

\[ a_B = 2.06 \text{ m/s}^2 \]

(b) For runner B:

\[ v_B = v_0 + a_B (t - t_B) \]

where \( t_B \) is the time at which he begins to run.

At \( t = 1.82 \) s:

\[ 9.078 \text{ m/s} = (2.0603 \text{ m/s}^2)(1.82 - t_B) \]

or

\[ t_B = -2.59 \text{ s} \]

Runner B should start to run 2.59 s before A reaches the exchange zone.
PROBLEM 11.41

Automobiles A and B are traveling in adjacent highway lanes and at \( t = 0 \) have the positions and speeds shown. Knowing that automobile A has a constant acceleration of 0.54 and that B has a constant deceleration of 0.36 determine (a) when and where A will overtake B, (b) the speed of each automobile at that time.

SOLUTION

\[ a_A = +0.54 \text{ m/s}^2 \quad a_B = -0.36 \text{ m/s}^2 \]

\[ |v_A|_0 = 36 \text{ km/h} = 10 \text{ m/s} \]
\[ |v_B|_0 = 54 \text{ km/h} = 15 \text{ m/s} \]

Motion of auto A:

\[ v_A = (v_A)_0 + a_A t = 10 + 0.54 t \quad (1) \]
\[ x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2 = 0 + 10t + \frac{1}{2}(0.54)t^2 \quad (2) \]

Motion of auto B:

\[ v_B = (v_B)_0 + a_B t = 15 - 0.36 t \quad (3) \]
\[ x_B = (x_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2 = 22.5 + 15t + \frac{1}{2}(-0.36)t^2 \quad (4) \]

(a) A overtakes B at \( t = t_1 \).

\[ x_A = x_B: \quad 10t_1 + 0.27t_1^2 = 22.5 + 15t_1 - 0.18t_1^2 \]
\[ 0.45t_1^2 - 5t_1 - 22.5 = 0 \]
\[ t_1 = -3.437 \text{ s} \quad \text{and} \quad t_1 = 14.548 \text{ s} \]
\[ t_1 = 14.548 \text{ s} \]

Eq. (2):

\[ x_A = 10(14.548) + 0.27(14.548)^2 \]
\[ x_A = 0.203 \text{ m} \]

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PROBLEM 11.41 (Continued)

(b) Velocities when $t_1 = 14.548$ s

Eq. (1):

$v_A = 10 + (0.54)(14.548)$
$v_A = 17.856 \text{ m/s}$
$v_A = 64.3 \text{ km/h}$ → ◄

Eq. (3):

$v_B = 15 - 0.36(14.548)$
$v_B = 9.7627 \text{ m/s}$
$v_B = 35.1 \text{ km/h}$ → ◄
PROBLEM 11.42

In a boat race, boat A is leading boat B by 40 m and both boats are traveling at a constant speed of 160 km/h. At $t = 0$, the boats accelerate at constant rates. Knowing that when B passes A, $t = 8$ s and $v_A = 200$ km/h, determine (a) the acceleration of A, (b) the acceleration of B.

SOLUTION

(a) We have

$$v_A = (v_A)_0 + a_A t$$

$$(v_A)_0 = 160 \text{ km/h} = \frac{400}{9} \text{ m/s}$$

At $t = 8$ s:

$$v_A = 200 \text{ km/h} = \frac{500}{9} \text{ m/s}$$

Then

$$\frac{500}{9} \text{ m/s} = \frac{400}{9} \text{ m/s} + a_A (8 \text{ s})$$

or

$$a_A = \frac{25}{18} \text{ m/s}^2$$

$a_A = 1.39 \text{ m/s}^2$ $\blacklozenge$

(b) We have

$$x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2$$

and

$$x_B = 0 + (v_B)_0 t + \frac{1}{2} a_B t^2$$

At $t = 8$ s:

$$x_A = x_B$$

$$40 \text{ m} + \left( \frac{400}{9} \text{ m/s} \right) (8 \text{ s}) + \frac{1}{2} \left( \frac{25}{18} \text{ m/s}^2 \right) (8 \text{ s})^2$$

$$= \left( \frac{400}{9} \text{ m/s} \right) (8 \text{ s}) + \frac{1}{2} a_B (8 \text{ s})^2$$

or

$$a_B = 2.64 \text{ m/s}^2$$ $\blacklozenge$
PROBLEM 11.43

Boxes are placed on a chute at uniform intervals of time \( t_R \) and slide down the chute with uniform acceleration. Knowing that as any box B is released, the preceding box A has already slid 6 m and that 1 s later they are 10 m apart, determine (a) the value of \( t_R \), (b) the acceleration of the boxes.

SOLUTION

Let \( t_s = 1 \) s be the time when the boxes are 10 m apart.

Let \( a_A = a_B = a; \ (x_A)_0 = (x_B)_0 = 0; \ (v_A)_0 = (v_B)_0 = 0. \)

(a) For \( t > 0, \ x_A = \frac{1}{2} at^2 \)

For \( t > t_R, \ x_B = \frac{1}{2} a(t - t_R)^2 \)

At \( t = t_R, \ x_A = 6 \) m \[ 6 = \frac{1}{2} at_R^2 \]

At \( t = t_R + t_s, \ x_A - x_B = 10 \) m

\[ 10 = \frac{1}{2} a(t_R + t_s)^2 - \frac{1}{2} a(t_R + t_s - t_R)^2 \]

\[ = \frac{1}{2} at_R^2 + at_R t_s + \frac{1}{2} at_R^2 - \frac{1}{2} at_s^2 = 18 + at_R t_s \]

\[ at_R = \frac{10 - 6}{t_s} = \frac{4}{1} = 4 \text{ m/s} \]

(2)

Dividing Equation (1) by Eq. (2),

\[ \frac{\frac{1}{2} at_R^2}{at_R} = \frac{1}{2} t_R = \frac{6}{4} \]

\[ t_R = 3.00 \text{ s} \]

(b) Solving Eq. (2) for \( a, \)

\[ a = \frac{4}{1} = 4 \text{ m/s}^2 \]

\[ a = 4 \text{ m/s}^2 \]
**PROBLEM 11.44**

Two automobiles A and B are approaching each other in adjacent highway lanes. At \( t = 0 \), A and B are 1 km apart, their speeds are \( v_A = 108 \text{ km/h} \) and \( v_B = 63 \text{ km/h} \), and they are at Points P and Q, respectively. Knowing that A passes Point Q 40 s after B was there and that B passes Point P 42 s after A was there, determine (a) the uniform accelerations of A and B, (b) when the vehicles pass each other, (c) the speed of B at that time.

---

**SOLUTION**

(a) We have

\[
x_A = 0 + (v_A)_0 t + \frac{1}{2} a_A t^2 \quad (v_A)_0 = 108 \text{ km/h} = 30 \text{ m/s}
\]

At \( t = 40 \text{ s} \):

\[
1000 \text{ m} = (30 \text{ m/s})(40 \text{ s}) + \frac{1}{2} a_A (40 \text{ s})^2
\]

or

\[
a_A = -0.250 \text{ m/s}^2
\]

Also,

\[
x_B = 0 + (v_B)_0 t + \frac{1}{2} a_B t^2 \quad (v_B)_0 = 63 \text{ km/h} = 17.5 \text{ m/s}
\]

At \( t = 42 \text{ s} \):

\[
1000 \text{ m} = (17.5 \text{ m/s})(42 \text{ s}) + \frac{1}{2} a_B (42 \text{ s})^2
\]

or

\[
a_B = 0.30045 \text{ m/s}^2
\]

(b) When the cars pass each other

\[
x_A + x_B = 1000 \text{ m}
\]

Then

\[
(30 \text{ m/s})t_{AB} + \frac{1}{2} (-0.250 \text{ m/s}^2)t_{AB}^2 + (17.5 \text{ m/s})t_{AB}
\]

\[
+ \frac{1}{2} (0.30045 \text{ m/s}^2)t_{AB}^2 = 1000 \text{ m}
\]

or

\[
0.05045t_{AB}^2 + 95t_{AB} - 2000 = 0
\]

Solving

\[
t = 20.822 \text{ s} \quad \text{and} \quad t = -1904 \text{ s}
\]

\( t > 0 \Rightarrow t_{AB} = 20.8 \text{ s} \)
PROBLEM 11.44 (Continued)

(c) We have

\[ v_B = (v_B)_0 + a_B t \]

At \( t = t_{AB} \):

\[ v_B = 17.5 \text{ m/s} + (0.30045 \text{ m/s}^2)(20.822 \text{ s}) \]
\[ = 23.756 \text{ m/s} \]

or

\[ v_B = 85.5 \text{ km/h} \]
**PROBLEM 11.45**

Car A is parked along the northbound lane of a highway, and car B is traveling in the southbound lane at a constant speed of 90 km/h. At $t = 0$, A starts and accelerates at a constant rate $a_A$, while at $t = 5$ s, B begins to slow down with a constant deceleration of magnitude $a_A/6$. Knowing that when the cars pass each other $x = 90$ m and $v_A = v_B$, determine (a) the acceleration $a_A$, (b) when the vehicles pass each other, (c) the distance $d$ between the vehicles at $t = 0$.

---

**SOLUTION**

For $t \geq 0$:

$$v_A = 0 + a_A t$$

$$x_A = 0 + 0 + \frac{1}{2} a_A t^2$$

$0 \leq t < 5$ s:

$$x_B = 0 + (v_B)_0 t \quad (v_B)_0 = 90 \text{ km/h} = 25 \text{ m/s}$$

At $t = 5$ s:

$$x_B = (25 \text{ m/s})(5 \text{ s}) = 125 \text{ m}$$

For $t \geq 5$ s:

$$v_B = (v_B)_0 + a_B (t - 5) \quad a_B = -\frac{1}{6} a_A$$

$$x_B = (x_B)_0 + (v_B)_0 (t - 5) + \frac{1}{2} a_B (t - 5)^2$$

Assume $t > 5$ s when the cars pass each other.

At that time ($t_{AB}$),

$v_A = v_B$:

$$a_A t_{AB} = (25 \text{ m/s}) - \frac{a_A}{6} (t_{AB} - 5)$$

$x_A = 90$ m:

$$90 \text{ m} = \frac{1}{2} a_A t_{AB}^2$$

---

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PROBLEM 11.45 (Continued)

Then
\[
\frac{a_A (\frac{2}{3} t_{AB} - \frac{5}{3})}{t_{AB}^2} = \frac{25}{90}
\]

or
\[
25t_{AB}^2 - 210t_{AB} + 150 = 0
\]

Solving
\[
t_{AB} = 0.788 \text{s and } t_{AB} = 7.6117 \text{s}
\]

(a) With \( t_{AB} > 5 \text{s} \),
\[
90 = \frac{1}{2} a_A (7.6117)^2
\]

or
\[
a_A = 3.10677 \text{ m/s}^2
\]

(b) From above
\[
t_{AB} = 7.61\text{s}
\]

Note: An acceptable solution cannot be found if it is assumed that \( t_{AB} \leq 5 \text{s} \).

(c) We have
\[
d = x + (x_B)_{t_{AB}}
\]

\[
= 90 + [125 + 25 m/s (7.6117 - 5) + \left(\frac{1}{2}\right) \left(\frac{-3.10677}{6}\right)(7.6117 - 5)^2]
\]

or
\[
d = 279 \text{ m}
\]
PROBLEM 11.46

Two blocks A and B are placed on an incline as shown. At \( t = 0 \), A is projected up the incline with an initial velocity of 8 m/s and B is released from rest. The blocks pass each other 1 s later, and B reaches the bottom of the incline when \( t = 3.4 \) s. Knowing that the maximum distance from the bottom of the incline reached by block A is 7 m and that the accelerations of A and B (due to gravity and friction) are constant and are directed down the incline, determine (a) the accelerations of A and B, (b) the distance \( d \), (c) the speed of A when the blocks pass each other.

SOLUTION

(a) We have

\[
\begin{align*}
\nu_A^2 &= (\nu_A)_0^2 + 2a_A(x_A - 0) \\
\text{When} \\
x_A &= (x_A)_{\text{max}}, \quad \nu_A = 0 \\
\text{Then} \\
0 &= (8 \text{ m/s})^2 + 2a_A(7 \text{ m}) \\
or \\
a_A &= \frac{-32}{7} \text{ m/s}^2 \\
or \\
Now \\
x_A &= 0 + (\nu_A)_0t + \frac{1}{2}a_At^2 \\
\text{and} \\
x_B &= 0 + 0t + \frac{1}{2}a_Bt^2
\end{align*}
\]

At \( t = 1 \) s, the blocks pass each other.

\[
(x_A)_1 + (x_B)_1 = d
\]

At \( t = 3.4 \) s, \( x_B = d \): 

Thus

\[
(x_A)_1 + (x_B)_1 = (x_B)_{3.4}
\]

or

\[
\left[ (8 \text{ m/s})(1 \text{s}) + \frac{1}{2}\left(\frac{-32 \text{ m}}{7 \text{ s}^2}\right)(1 \text{s})^2 \right] + \left[ \frac{1}{2}a_B(1 \text{s})^2 \right] = \frac{1}{2}a_B(3.4 \text{ s})^2
\]

or

\[
a_B = 1.08225 \text{ m/s}^2 \\
a_B = 1.082 \text{ m/s}^2
\]
PROBLEM 11.46 (Continued)

(b) At \( t = 3.4 \text{ s} \), \( x_B = d \):
\[
d = \frac{1}{2}(1.08225 \text{ m/s}^2)(3.4 \text{ s})^2
\]
or
\[
d = 6.26 \text{ m}
\]

(c) We have
\[
v_A = (v_A)_0 + a_A t
\]
At \( t = 1 \text{ s} \):
\[
v_A = 8 \text{ m/s} + \left(-\frac{32}{7} \text{ m/s}^2\right)(1 \text{ s})
\]
or
\[
v_A = 3.43 \text{ m/s}
\]
PROBLEM 11.47

Slider block A moves to the left with a constant velocity of 6 m/s. Determine (a) the velocity of block B, (b) the velocity of portion D of the cable, (c) the relative velocity of portion C of the cable with respect to portion D.

SOLUTION

From the diagram, we have

\[ x_A + 3y_B = \text{constant} \]

Then

\[ v_A + 3v_B = 0 \]  \hspace{1cm} (1)

and

\[ a_A + 3a_B = 0 \]  \hspace{1cm} (2)

(a) Substituting into Eq. (1) \[ 6 \text{ m/s} + 3v_B = 0 \]

or

\[ v_B = 2 \text{ m/s} \uparrow \]

(b) From the diagram \[ y_B + y_D = \text{constant} \]

Then

\[ v_B + v_D = 0 \]

\[ v_D = 2 \text{ m/s} \downarrow \]

(c) From the diagram \[ x_A + y_C = \text{constant} \]

Then

\[ v_A + v_C = 0 \]

\[ v_C = -6 \text{ m/s} \]

Now \[ v_{C/D} = v_C - v_D = (-6 \text{ m/s}) - (2 \text{ m/s}) = -8 \text{ m/s} \]

\[ v_{C/D} = 8 \text{ m/s} \uparrow \]
**PROBLEM 11.48**

Block B starts from rest and moves downward with a constant acceleration. Knowing that after slider block A has moved 400 mm its velocity is 4 m/s, determine (a) the accelerations of A and B, (b) the velocity and the change in position of B after 2 s.

**SOLUTION**

From the diagram, we have

\[ x_A + 3y_B = \text{constant} \]

Then

\[ v_A + 3v_B = 0 \] (1)

and

\[ a_A + 3a_B = 0 \] (2)

(a) Eq. (2): \( a_A + 3a_B = 0 \) and \( a_B \) is constant and positive \( \Rightarrow a_A \) is constant and negative

Also, Eq. (1) and \( (v_B)_0 = 0 \Rightarrow (v_A)_0 = 0 \)

Then \[ v_A^2 = 0 + 2a_A[x_A - (x_A)_0] \]

When \(|\Delta x_A| = 0.4 \text{ m}: \]

\[ (4 \text{ m/s})^2 = 2a_A(0.4 \text{ m}) \]

or \[ a_A = 20 \text{ m/s}^2 \]

Then, substituting into Eq. (2):

\[ -20 \text{ m/s}^2 + 3a_B = 0 \]

or \[ a_B = \frac{20}{3} \text{ m/s}^2 \]

\[ a_B = 6.67 \text{ m/s}^2 \]
PROBLEM 11.48 (Continued)

(b) We have

\[ v_B = 0 + a_B t \]

At \( t = 2 \) s:

\[ v_B = \left( \frac{20}{3} \text{ m/s}^2 \right) (2 \text{ s}) \]

or

\[ v_B = 13.33 \text{ m/s} \downarrow \]

Also

\[ y_B = (y_B)_0 + \frac{1}{2} a_B t^2 \]

At \( t = 2 \) s:

\[ y_B - (y_B)_0 = \frac{1}{2} \left( \frac{20}{3} \text{ m/s}^2 \right) (2 \text{ s})^2 \]

or

\[ y_B - (y_B)_0 = 13.33 \text{ m} \downarrow \]
**PROBLEM 11.49**

The elevator shown in the figure moves downward with a constant velocity of 5 m/s. Determine (a) the velocity of the cable C, (b) the velocity of the counterweight W, (c) the relative velocity of the cable C with respect to the elevator, (d) the relative velocity of the counterweight W with respect to the elevator.

**SOLUTION**

Choose the positive direction downward.

(a) Velocity of cable C.

\[ y_C + 2y_E = \text{constant} \]
\[ v_C + 2v_E = 0 \]

But,
\[ v_E = 5 \text{ m/s} \]
or
\[ v_C = -2v_E = -10 \text{ m/s} \]

\[ \mathbf{v}_C = 10.00 \text{ m/s} \uparrow \]

(b) Velocity of counterweight W.

\[ y_W + y_E = \text{constant} \]
\[ v_W + v_E = 0 \]
\[ v_W = -v_E = -5 \text{ m/s} \]

\[ \mathbf{v}_W = 5.00 \text{ m/s} \uparrow \]

(c) Relative velocity of C with respect to E.

\[ v_{C/E} = v_C - v_E = (-10 \text{ m/s}) - (+5 \text{ m/s}) = -15 \text{ m/s} \]

\[ \mathbf{v}_{C/E} = 15.00 \text{ m/s} \uparrow \]

(d) Relative velocity of W with respect to E.

\[ v_{W/E} = v_W - v_E = (-5 \text{ m/s}) - (5 \text{ m/s}) = -10 \text{ m/s} \]

\[ \mathbf{v}_{W/E} = 10.00 \text{ m/s} \uparrow \]
PROBLEM 11.50

The elevator shown starts from rest and moves upward with a constant acceleration. If the counterweight $W$ moves through 10 m in 5 s, determine (a) the accelerations of the elevator and the cable $C$, (b) the velocity of the elevator after 5 s.

SOLUTION

We choose Positive direction downward for motion of counterweight.

\[ y_w = \frac{1}{2} a_w t^2 \]

At \( t = 5 \text{ s} \),

\[ y_w = 10 \text{ m} \]

\[ 10 \text{ m} = \frac{1}{2} a_w (5 \text{ s})^2 \]

\[ a_w = 0.8 \text{ m/s}^2 \]

(a) Accelerations of $E$ and $C$.

Since \( y_w + y_E = \text{constant} \), \( v_w + v_E = 0 \), and \( a_w + a_E = 0 \)

Thus:

\[ a_E = -a_w = -(0.8 \text{ m/s}^2) \],

\[ a_E = 0.8 \text{ m/s}^2 \uparrow \]

Also,

\[ y_C + 2y_E = \text{constant} \], \( v_C + 2v_E = 0 \), and \( a_C + 2a_E = 0 \)

Thus:

\[ a_C = -2a_E = -2(-0.8 \text{ m/s}^2) = 1.6 \text{ m/s}^2 \],

\[ a_C = 1.6 \text{ m/s}^2 \downarrow \]

(b) Velocity of elevator after 5 s.

\[ v_E = (v_E)_0 + a_E t = 0 + (-0.8 \text{ m/s}^2)(5 \text{ s}) = -4 \text{ m/s} \]

\[ (v_E)_5 = 4.00 \text{ m/s} \uparrow \]
**PROBLEM 11.51**

Collar A starts from rest and moves upward with a constant acceleration. Knowing that after 8 s the relative velocity of collar B with respect to collar A is 0.6 m/s, determine (a) the accelerations of A and B, (b) the velocity and the change in position of B after 6 s.

**SOLUTION**

From the diagram

\[ 2y_A + y_B + (y_B - y_A) = \text{constant} \]

Then

\[ v_A + 2v_B = 0 \quad (1) \]

and

\[ a_A + 2a_B = 0 \quad (2) \]

(a) Eq. (1) and \( (v_A)_0 = 0 = (v_B)_0 \)

Also, Eq. (2) and \( a_A \) is constant and negative \( \Rightarrow a_B \) is constant and positive

Then

\[ v_A = 0 + a_A t \quad v_B = 0 + a_B t \]

Now

\[ v_{B/A} = v_B - v_A = (a_B - a_A)t \]

From Eq. (2)

\[ a_B = -\frac{1}{2} a_A \]

So that

\[ v_{B/A} = -\frac{3}{2} a_A t \]
PROBLEM 11.51 (Continued)

At $t = 8\text{ s}$:

$$0.6\text{ m/s} = -\frac{3}{2} a_A(8\text{ s})$$

or

$$a_A = -\frac{1}{20}\text{ m/s}^2$$

and then

$$a_B = -\frac{1}{2}\left(-\frac{1}{20}\text{ m/s}^2\right) = \frac{1}{40}\text{ m/s}^2$$

or

$$a_B = 0.250\text{ m/s}^2$$

(b) At $t = 6\text{ s}$:

$$v_B = \left(\frac{1}{40}\text{ m/s}^2\right)(6\text{ s}) = 0.15\text{ m/s}$$

or

$$v_B = 0.1500\text{ m/s}$$

Now

$$y_B = (y_B)_0 + 0 + \frac{1}{2} a_B t^2$$

At $t = 6\text{ s}$:

$$y_B - (y_B)_0 = \frac{1}{2}\left(\frac{1}{40}\text{ m/s}^2\right)(6\text{ s})^2 = 0.45\text{ m}$$

or

$$y_B - (y_B)_0 = 0.450\text{ m}$$
**PROBLEM 11.52**

In the position shown, collar B moves downward with a velocity of 0.3 m/s. Determine (a) the velocity of collar A, (b) the velocity of portion C of the cable, (c) the relative velocity of portion C of the cable with respect to collar B.

**SOLUTION**

From the diagram

\[2y_A + y_B = (y_B - y_A) = \text{constant}\]

Then

\[v_A + 2v_B = 0 \quad (1)\]

and

\[a_A + 2a_B = 0 \quad (2)\]

(a) Substituting into Eq. (1)

\[v_A + 2(0.3 \text{ m/s}) = 0\]

or

\[v_A = 0.600 \text{ m/s} \uparrow\]

(b) From the diagram

\[2y_A + y_C = \text{constant}\]

Then

\[2v_A + v_C = 0\]

Substituting

\[2(-0.6 \text{ m/s}) + v_C = 0\]

or

\[v_C = 1.200 \text{ m/s} \downarrow\]

(c) We have

\[v_{C/B} = v_C - v_B\]

\[= (1.2 \text{ m/s}) - (0.3 \text{ m/s})\]

or

\[v_{C/B} = 0.900 \text{ m/s} \downarrow\]
**PROBLEM 11.53**

Slider block B moves to the right with a constant velocity of 300 mm/s. Determine (a) the velocity of slider block A, (b) the velocity of portion C of the cable, (c) the velocity of portion D of the cable, (d) the relative velocity of portion C of the cable with respect to slider block A.

**SOLUTION**

From the diagram

\[ x_B + (x_B - x_A) - 2x_A = \text{constant} \]

Then

\[ 2v_B - 3v_A = 0 \]  \hspace{1cm} (1)

and

\[ 2a_B - 3a_A = 0 \]  \hspace{1cm} (2)

Also, we have

\[ -x_B - x_A = \text{constant} \]

Then

\[ v_B + v_A = 0 \]  \hspace{1cm} (3)

(a) Substituting into Eq. (1)

\[ 2(300 \text{ mm/s}) - 3v_A = 0 \]

or

\[ v_A = 200 \text{ mm/s} \rightarrow \]

(b) From the diagram

\[ x_B + (x_B - x_C) = \text{constant} \]

Then

\[ 2v_B - v_C = 0 \]

Substituting

\[ 2(300 \text{ mm/s}) - v_C = 0 \]

or

\[ v_C = 600 \text{ mm/s} \rightarrow \]

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PROBLEM 11.53 (Continued)

(c) From the diagram  \((x_C - x_A) + (x_B - x_A) = \text{constant}\)

Then  \(v_C - 2v_A + v_D = 0\)

Substituting  \(600 \text{ mm/s} - 2(200 \text{ mm/s}) + v_D = 0\)

or  \(v_D = 200 \text{ mm/s} \leftarrow \downarrow\)

(d) We have  \(v_{C/A} = v_C - v_A\)

\[= 600 \text{ mm/s} - 200 \text{ mm/s}\]

or  \(v_{C/A} = 400 \text{ mm/s} \rightarrow \downarrow\)
PROBLEM 11.54

At the instant shown, slider block B is moving with a constant acceleration, and its speed is 150 mm/s. Knowing that after slider block A has moved 240 mm to the right its velocity is 60 mm/s, determine (a) the accelerations of A and B, (b) the acceleration of portion D of the cable, (c) the velocity and change in position of slider block B after 4 s.

SOLUTION

From the diagram

\[ x_B + (x_B - x_A) - 2x_A = \text{constant} \]

Then

\[ 2v_B - 3v_A = 0 \quad (1) \]

and

\[ 2a_B - 3a_A = 0 \quad (2) \]

(a) First observe that if block A moves to the right, \( v_A \rightarrow \) and Eq. (1) \( \Rightarrow v_B \rightarrow \). Then, using Eq. (1) at \( t = 0 \)

\[ 2(150 \text{ mm/s}) - 3(v_A)_0 = 0 \]

or

\[ (v_A)_0 = 100 \text{ mm/s} \]

Also, Eq. (2) and \( a_B = \text{constant} \Rightarrow a_A = \text{constant} \)

Then

\[ v_A^2 = (v_A)_0^2 + 2a_A(x_A - (x_A)_0) \]

When \( x_A - (x_A)_0 = 240 \text{ mm} \):

\[ (60 \text{ mm/s})^2 = (100 \text{ mm/s})^2 + 2a_A(240 \text{ mm}) \]

or

\[ a_A = -\frac{40}{3} \text{ mm/s}^2 \]

or

\[ a_A = 13.33 \text{ mm/s}^2 \]

\[ \rightarrow \]
PROBLEM 11.54 (Continued)

Then, substituting into Eq. (2)

\[ 2a_B - 3\left(\frac{-40}{3} \text{ mm/s}^2\right) = 0 \]

or \[ a_B = -20 \text{ mm/s}^2 \quad \text{or} \quad a_B = 20.0 \text{ mm/s}^2 \leftrightarrow \square \]

(b) From the solution to Problem 11.53

\[ v_D + v_A = 0 \]

Then \[ a_D + a_A = 0 \]

Substituting \[ a_B + \left(\frac{-40}{3} \text{ mm/s}^2\right) = 0 \]

or \[ a_B = 13.33 \text{ mm/s}^2 \rightarrow \square \]

(c) We have \[ v_B = (v_B)_0 + a_B t \]

At \( t = 4 \) s:

\[ v_B = 150 \text{ mm/s} + (-20.0 \text{ mm/s}^2)(4 \text{ s}) \]

or \[ v_B = 70.0 \text{ mm/s} \rightarrow \square \]

Also \[ y_B = (y_B)_0 + (v_B)_0 t + \frac{1}{2}a_B t^2 \]

At \( t = 4 \) s:

\[ y_B - (y_B)_0 = (150 \text{ mm/s})(4 \text{ s}) + \frac{1}{2}(-20.0 \text{ mm/s}^2)(4 \text{ s})^2 \]

or \[ y_B - (y_B)_0 = 440 \text{ mm} \rightarrow \square \]
PROBLEM 11.55

Block B moves downward with a constant velocity of 20 mm/s. At t = 0, block A is moving upward with a constant acceleration, and its velocity is 30 mm/s. Knowing that at t = 3 s slider block C has moved 57 mm to the right, determine (a) the velocity of slider block C at t = 0, (b) the accelerations of A and C, (c) the change in position of block A after 5 s.

SOLUTION

From the diagram

\[ 3y_A + 4y_B + x_C = \text{constant} \]

Then

\[ 3v_A + 4v_B + v_C = 0 \]  \( \text{(1)} \)

and

\[ 3a_A + 4a_B + a_C = 0 \]  \( \text{(2)} \)

Given:

\[ v_B = 20 \text{ mm/s} \downarrow; \]
\[ (v_A)_0 = 30 \text{ mm/s} \uparrow \]

(a) Substituting into Eq. (1) at t = 0

\[ 3(-30 \text{ mm/s}) + 4(20 \text{ mm/s}) + (v_C)_0 = 0 \]

\[ v_C = 10 \text{ mm/s} \quad \text{or} \quad (v_C)_0 = 10 \text{ mm/s} \rightarrow \]

(b) We have

\[ x_C = (x_C)_0 + (v_C)_0 t + \frac{1}{2} a_C t^2 \]

At t = 3 s:

\[ 57 \text{ mm} = (10 \text{ mm/s})(3 \text{ s}) + \frac{1}{2} a_C (3 \text{ s})^2 \]

\[ a_C = 6 \text{ mm/s}^2 \quad \text{or} \quad a_C = 6 \text{ mm/s}^2 \rightarrow \]

Now

\[ v_B = \text{constant} \rightarrow a_B = 0 \]
Then, substituting into Eq. (2)
\[ 3a_A + 4(0) + (6 \text{ mm/s}^2) = 0 \]
\[ a_A = -2 \text{ mm/s}^2 \quad \text{or} \quad a_A = 2 \text{ mm/s}^2 \]

(c) We have
\[ y_A = (y_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2 \]
At \( t = 5 \text{ s} \):
\[ y_A - (y_A)_0 = (-30 \text{ mm/s})(5 \text{ s}) + \frac{1}{2} (-2 \text{ mm/s}^2)(5 \text{ s})^2 \]
\[ = -175 \text{ mm} \]
\[ y_A - (y_A)_0 = 175 \text{ mm} \]
**PROBLEM 11.56**

Block B starts from rest, block A moves with a constant acceleration, and slider block C moves to the right with a constant acceleration of $75 \text{ mm/s}^2$. Knowing that at $t = 2 \text{ s}$ the velocities of B and C are $480 \text{ mm/s}$ downward and $280 \text{ mm/s}$ to the right, respectively, determine (a) the accelerations of A and B, (b) the initial velocities of A and C, (c) the change in position of slider C after $3 \text{ s}$.

**SOLUTION**

From the diagram

$$3y_A + 4y_B + x_C = \text{constant}$$

Then

$$3v_A + 4v_B + v_C = 0 \quad (1)$$

and

$$3a_A + 4a_B + a_C = 0 \quad (2)$$

**Given:**

$$(v_B) = 0, \quad a_B = \text{constant}$$

$$(a_C) = 75 \text{ mm/s}^2 \rightarrow$$

At $t = 2 \text{ s}$,

$$v_B = 480 \text{ mm/s} \downarrow$$

$$v_C = 280 \text{ mm/s} \rightarrow$$

(a) Eq. (2) and $a_A = \text{constant}$ and $a_C = \text{constant} \Rightarrow a_B = \text{constant}$

Then

$$v_B = 0 + a_B t$$

At $t = 2 \text{ s}$:

$$480 \text{ mm/s} = a_B (2 \text{ s})$$

$$a_B = 240 \text{ mm/s}^2 \quad \text{or} \quad a_B = 240 \text{ mm/s}^2 \downarrow \blacktriangledown$$

Substituting into Eq. (2)

$$3a_A + 4(240 \text{ mm/s}^2) + (75 \text{ mm/s}^2) = 0$$

$$a_A = -345 \text{ mm/s} \quad \text{or} \quad a_A = 345 \text{ mm/s}^2 \uparrow \blacktriangledown$$
PROBLEM 11.56 (Continued)

(b) We have
\[ v_C = (v_C)_0 + a_C t \]
At \( t = 2 \) s:
\[ 280 \text{ mm/s} = (v_C)_0 + (75 \text{ mm/s})(2 \text{ s}) \]
\[ v_C = -130 \text{ mm/s} \quad \text{or} \quad (v_C)_0 = 130 \text{ mm/s} \rightarrow \]
Then, substituting into Eq. (1) at \( t = 0 \)
\[ 3(v_A)_0 + 4(0) + (130 \text{ mm/s}) = 0 \]
\[ v_A = -43.3 \text{ mm/s} \quad \text{or} \quad (v_A)_0 = 43.3 \text{ mm/s} \uparrow \]

(c) We have
\[ x_C = (x_C)_0 + (v_C)_0 t + \frac{1}{2} a_C t^2 \]
At \( t = 3 \) s:
\[ x_C - (x_C)_0 = (130 \text{ mm/s})(3 \text{ s}) + \frac{1}{2} (75 \text{ mm/s}^2)(3 \text{ s})^2 \]
\[ = -728 \text{ mm} \quad \text{or} \quad x_C - (x_C)_0 = 728 \text{ mm} \rightarrow \]
**PROBLEM 11.57**

Collar A starts from rest at \( t = 0 \) and moves downward with a constant acceleration of 175 mm/s\(^2\). Collar B moves upward with a constant acceleration, and its initial velocity is 200 mm/s. Knowing that collar B moves through 500 mm between \( t = 0 \) and \( t = 2 \) s, determine (a) the accelerations of collar B and block C, (b) the time at which the velocity of block C is zero, (c) the distance through which block C will have moved at that time.

**SOLUTION**

From the diagram

\[-y_A + (y_C - y_A) + 2y_C + (y_C - y_B) = \text{constant}\]

Then

\[-2v_A - v_B + 4v_C = 0 \tag{1}\]

and

\[-2a_A - a_B + 4a_C = 0 \tag{2}\]

Given:

- \((v_A)_0 = 0\)
- \((a_A) = 175 \text{ mm/s}^2\) \(\downarrow\)
- \((v_B)_0 = 200 \text{ mm/s} \uparrow\)
- \(a_B = \text{constant} \uparrow\)

At \( t = 2 \) s

\[y - (y_B)_0 = 500 \text{ mm} \uparrow\]

(a) We have

\[y_B = (y_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2\]

At \( t = 2 \) s:

\[-500 \text{ mm} = (-200 \text{ mm/s})(2 \text{ s}) + \frac{1}{2} a_B (2 \text{ s})^2\]

\[a_B = -50 \text{ mm/s}^2 \text{ or } a_B = 50.0 \text{ mm/s}^2 \uparrow \uparrow\]

Then, substituting into Eq. (2)

\[-2(175 \text{ mm/s}^2) - (-50 \text{ mm/s}^2) + 4a_C = 0\]

\[a_C = 75 \text{ mm/s}^2 \text{ or } a_C = 75.0 \text{ mm/s}^2 \downarrow\]
PROBLEM 11.57 (Continued)

(b) Substituting into Eq. (1) at \( t = 0 \)

\[-2(0) - (-200 \text{ mm/s}) + 4(v_C)_0 = 0 \quad \text{or} \quad (v_C)_0 = -50 \text{ mm/s} \]

Now

\[v_C = (v_C)_0 + a_C t\]

When \( v_C = 0: \)

\[0 = (-50 \text{ mm/s}) + (75 \text{ mm/s}^2) t\]

or

\[t = \frac{2}{3} \text{ s} \quad \text{ or } \quad t = 0.667 \text{ s} \]

(c) We have

\[y_C = (y_C)_0 + (v_C)_0 t + \frac{1}{2} a_C t^2\]

At \( t = \frac{2}{3} \text{ s} \):

\[y_C - (y_C)_0 = (-50 \text{ mm/s}) \left( \frac{2}{3} \text{ s} \right) + \frac{1}{2} (75 \text{ mm/s}^2) \left( \frac{2}{3} \text{ s} \right)^2\]

\[= -\frac{50}{3} \text{ mm} = -16.667 \text{ mm} \quad \text{or} \quad y_C - (y_C)_0 = 16.67 \text{ mm} \uparrow \]
**PROBLEM 11.58**

Collars A and B start from rest, and collar A moves upward with an acceleration of $75t^2 \text{ mm/s}^2$. Knowing that collar B moves downward with a constant acceleration and that its velocity is 200 mm/s after moving 800 mm, determine (a) the acceleration of block C, (b) the distance through which block C will have moved after 3 s.

**SOLUTION**

From the diagram

$$-y_A + (y_C - y_A) + 2y_C + (y_C - y_B) = \text{constant}$$

Then

$$-2v_A - v_B + 4v_C = 0 \quad (1)$$

and

$$-2a_A - a_B + 4a_C = 0 \quad (2)$$

Given:

- $(v_A)_0 = 0$
- $(v_B)_0 = 0$
- $a_A = 75t^2 \text{ mm/s}^2 \uparrow$
- $a_B = \text{constant} \downarrow$

When $y_B - (y_B)_0 = 800 \text{ mm} \downarrow$, $v_B = 200 \text{ mm/s}$

(a) We have

$$v_B^2 = 0 + 2a_B[y_B - (y_B)_0]$$

When $y_B - (y_B)_0 = 800 \text{ mm}$:

$$200^2 \text{ mm/s}^2 = 2a_B(800 \text{ mm})$$

or

$$a_B = 25 \text{ mm/s}^2$$

Then, substituting into Eq. (2)

$$-2(-75t^2 \text{ mm/s}^2) - (25 \text{ mm/s}^2) + 4a_C = 0$$

or

$$a_C = -\frac{1}{4}(25 - 150t^2) \text{ mm/s}^2 \Rightarrow$$

(b) Substituting into Eq. (1) at $t = 0$

$$-2(0) - (0) + 4(v_C)_0 = 0 \quad \text{or} \quad (v_C)_0 = 0$$

Now

$$\frac{dv_C}{dt} = a_C = \frac{25}{4}(1 - 6t^2)$$
PROBLEM 11.58 (Continued)

At \( t = 0 \), \( v_C = 0 \):

\[
\int_0^{v_C} dv_C = \int_0^t \frac{25}{4} (1 - 2t^2) dt
\]

or

\[
v_C = \frac{25}{4} (t - 2t^3)
\]

Thus,

\[
v_C = 0
\]

At

\[
\frac{25}{4}(1 - 2t^2) = 0
\]

or

\[
t = 0, \quad t = \frac{1}{\sqrt{2}} \text{ s}
\]

Therefore, block C initially moves downward \( (v_C > 0) \) and then moves upward \( (v_C < 0) \).

Now

\[
\frac{dy_C}{dt} = v_C = \frac{25}{4} (t - 2t^3)
\]

At \( t = 0 \), \( y_C = (y_C)_0 \):

\[
\int_{(y_C)_0}^{y_C} dy_C = \int_0^t \frac{25}{4} (t - 2t^3) dt
\]

or

\[
y_C - (y_C)_0 = \frac{25}{8} (t^2 - t^4)
\]

At \( t = \frac{1}{\sqrt{2}} \text{ s} \):

\[
y_C - (y_C)_0 = \frac{25}{8} \left[ \left( \frac{1}{\sqrt{2}} \right)^2 - \left( \frac{1}{\sqrt{2}} \right)^4 \right] = \frac{25}{32} \text{ mm}
\]

At \( t = 3 \text{ s} \):

\[
y_C - (y_C)_0 = \frac{25}{8} [(3)^2 - (3)^4] = -225 \text{ mm}
\]

Total distance traveled

\[
= \left( \frac{25}{32} \right) + \left| -225 - \frac{25}{32} \right| = 226.5625 \text{ mm}
\]

\[
= 227 \text{ mm}
\]
**PROBLEM 11.59**

The system shown starts from rest, and each component moves with a constant acceleration. If the relative acceleration of block C with respect to collar B is 60 mm/s² upward and the relative acceleration of block D with respect to block A is 110 mm/s² downward, determine (a) the velocity of block C after 3 s, (b) the change in position of block D after 5 s.

**SOLUTION**

From the diagram

Cable 1: \[ 2y_A + 2y_B + y_C = \text{constant} \]

Then \[ 2v_A + 2v_B + v_C = 0 \] (1)

and \[ 2a_A + 2a_B + a_C = 0 \] (2)

Cable 2: \[ (y_D - y_A) + (y_D - y_B) = \text{constant} \]

Then \[ -v_A - v_B + 2v_D = 0 \] (3)

and \[ -a_A - a_B + 2a_D = 0 \] (4)

Given: At \( t = 0, \ v = 0 \); all accelerations constant;

\( a_{C,IB} = 60 \text{ mm/s}^2 \), \( a_{D/A} = 110 \text{ mm/s}^2 \)

(a) We have \( a_{C,IB} = a_C - a_B = -60 \) or \( a_B = a_C + 60 \)

and \( a_{D/A} = a_D - a_A = 110 \) or \( a_A = a_D - 110 \)

Substituting into Eqs. (2) and (4)

Eq. (2): \[ 2(a_D - 110) + 2(a_C + 60) + a_C = 0 \]

or \[ 3a_C + a_D = 100 \] (5)

Eq. (4): \[ -(a_D - 110) - (a_C + 60) + 2a_D = 0 \]

or \[ -a_C + a_D = -50 \] (6)
Solving Eqs. (5) and (6) for $a_C$ and $a_D$

$$a_C = 40 \text{ mm/s}^2$$
$$a_D = -10 \text{ mm/s}^2$$

Now

At $t = 3\,\text{s}$:

$$v_C = 0 + a_C t$$

or

At $t = 3\,\text{s}$:

$$v_C = (40 \text{ mm/s}^2)(3 \text{ s})$$

or

$$v_C = 120 \text{ mm/s}$$

(b) We have

$$y_D = (y_D)_0 + (0)t + \frac{1}{2}a_D t^2$$

At $t = 5\,\text{s}$:

$$y_D - (y_D)_0 = \frac{1}{2}(-10 \text{ mm/s}^2)(5 \text{ s})^2$$

or

$$y_D - (y_D)_0 = 125 \text{ mm}$$

**PROBLEM 11.60**

The system shown starts from rest, and the length of the upper cord is adjusted so that A, B, and C are initially at the same level. Each component moves with a constant acceleration, and after 2 s the relative change in position of block C with respect to block A is 280 mm upward. Knowing that when the relative velocity of collar B with respect to block A is 80 mm/s downward, the displacements of A and B are 160 mm downward and 320 mm downward, respectively, determine (a) the accelerations of A and B if \( a_B > 10 \text{ mm/s}^2 \), (b) the change in position of block D when the velocity of block C is 600 mm/s upward.

**SOLUTION**

From the diagram

Cable 1: \[ 2y_A + 2y_B + y_C = \text{constant} \]

Then

\[ 2v_A + 2v_B + v_C = 0 \]  \hspace{1cm} (1)

and \[ 2a_A + 2a_B + a_C = 0 \]  \hspace{1cm} (2)

Cable 2: \[ (y_D - y_A) + (y_D - y_B) = \text{constant} \]

Then

\[ -v_A - v_B - 2v_D = 0 \]  \hspace{1cm} (3)

and \[ -a_A - a_B + 2a_D = 0 \]  \hspace{1cm} (4)

Given: At \( t = 0 \)

\[ v = 0 \]

\[ (y_A)_0 = (y_B)_0 = (y_C)_0 \]

All accelerations constant at \( t = 2 \text{ s} \)

\[ y_{C/A} = 280 \text{ mm up} \]

When

\[ v_{B/A} = 80 \text{ mm/s down} \]

\[ y_A - (y_A)_0 = 160 \text{ mm up} \]

\[ y_B - (y_B)_0 = 320 \text{ mm down} \]

\[ a_B > 10 \text{ mm/s}^2 \]
**PROBLEM 11.60* (Continued)**

(a) We have

\[ y_A = (y_A)_0 + (0)t + \frac{1}{2}a_At^2 \]

and

\[ y_C = (y_C)_0 + (0)t + \frac{1}{2}a.Ct^2 \]

Then

\[ y_{C/A} = y_C - y_A = \frac{1}{2}(a_C - a_A)t^2 \]

At \( t = 2 \text{s} \), \( y_{C/A} = -280 \text{ mm} \):

\[-280 \text{ mm} = \frac{1}{2}(a_C - a_A)(2 \text{ s})^2 \]

or

\[ a_C = a_A - 140 \]

(5)

Substituting into Eq. (2)

\[ 2a_A + 2a_B + (a_A - 140) = 0 \]

or

\[ a_A = \frac{1}{3}(140 - 2a_B) \]

(6)

Now

\[ v_B = 0 + a_Bt \]

\[ v_A = 0 + a_At \]

\[ v_{B/A} = v_B - v_A = (a_B - a_A)t \]

Also

\[ y_B = (y_B)_0 + (0)t + \frac{1}{2}a_Bt^2 \]

When

\[ v_{B/A} = 80 \text{ mm/s} \downarrow : \ 80 = (a_B - a_A)t \]

(7)

\[ \Delta y_A = 160 \text{ mm} \downarrow : \ 160 = \frac{1}{2}a_A^2t^2 \]

\[ \Delta y_B = 320 \text{ mm} \downarrow : \ 320 = \frac{1}{2}a_B^2t^2 \]

Then

\[ 160 = \frac{1}{2}(a_B - a_A)t^2 \]

Using Eq. (7)

\[ 320 = (80)t \quad \text{or} \quad t = 4 \text{ s} \]

Then

\[ 160 = \frac{1}{2}a_A(4)^2 \quad \text{or} \quad a_A = 20 \text{ mm/s}^2 \downarrow \]

and

\[ 320 = \frac{1}{2}a_B(4)^2 \quad \text{or} \quad a_B = 40 \text{ mm/s}^2 \downarrow \]

Note that Eq. (6) is not used; thus, the problem is over-determined.
A different solution:
We have
\[ v_A^2 = (0) + 2a_A(y_A - (y_A)_0) \]
\[ v_B^2 = (0) + 2a_B(y_B - (y_B)_0) \]
Then
\[ v_{B/A} = v_B - v_A = \sqrt{2a_B(y_B - (y_B)_0)} - \sqrt{2a_A(y_A - (y_A)_0)} \]
When
\[ v_{B/A} = 80 \text{ mm/s} \downarrow: \]
\[ 80 \text{ mm/s} = \sqrt{2\left[ \sqrt{a_B(320 \text{ mm})} - \sqrt{a_A(160 \text{ mm})} \right]} \]
or
\[ 20 = \sqrt{2\left( \sqrt{200_B} - \sqrt{100_A} \right)} \tag{8} \]
Solving Eqs. (6) and (8) yields \( a_A \) and \( a_B \).

(b) Substituting into Eq. (5)
\[ a_C = 20 - 140 = -120 \text{ mm/s}^2 \]
and into Eq. (4)
\[-(20 \text{ mm/s}^2) - (40 \text{ mm/s}^2) + 2a_B = 0 \]
or
\[ a_B = 30 \text{ mm/s}^2 \]
Now
\[ v_C = 0 + a_C t \]
When \( v_C = -600 \text{ mm/s} \):
\[ -600 \text{ mm/s} = (-120 \text{ mm/s}^2)t \]
or
\[ t = 5 \text{ s} \]
Also
\[ y_D = (y_D)_0 + (0)t + \frac{1}{2}a_Bt^2 \]
At \( t = 5 \text{ s} \):
\[ y_D - (y_D)_0 = \frac{1}{2}(30 \text{ mm/s}^2)(5 \text{ s})^2 \]
or
\[ y_D - (y_D)_0 = 375 \text{ mm} \downarrow \]
PROBLEM 11.61

A subway car leaves station A; it gains speed at the rate of 4 m/s² for 6 s and then at the rate of 6 m/s² until it has reached the speed of 36 m/s. The car maintains the same speed until it approaches station B; brakes are then applied, giving the car a constant deceleration and bringing it to a stop in 6 s. The total running time from A to B is 40 s. Draw the a–t, v–t, and x–t curves, and determine the distance between stations A and B.

SOLUTION

Acceleration-Time Curve. Since the acceleration is either constant or zero, the a–t curve is made of horizontal straight-line segments. The values of a₂ and a₄ are determined as follows:

\[0 < t < 6: \quad \text{Change in } v = \text{area under } a–t \text{ curve}\]
\[v₆ - 0 = (6 \text{ s})(4 \text{ m/s}^2) = 24 \text{ m/s}\]

\[6 < t < t₂: \quad \text{Since the velocity increases from 24 m/s to 36 m/s,}
\[\text{Change in } v = \text{area under } a–t \text{ curve}\]
\[36 \text{ m/s} - 24 \text{ m/s} = (t₂ - 6)(6 \text{ m/s}^2) \quad t₂ = 8 \text{ s}\]

\[t₂ < t < 34: \quad \text{Since the velocity is constant the acceleration is zero.}\]

\[34 < t < 40: \quad \text{Change in } v = \text{area under } a–t \text{ curve}\]
\[0 - 36 \text{ m/s} = (6 \text{ s})a₄ \quad a₄ = -6 \text{ m/s}^2\]

The acceleration being negative, the corresponding area is below the t axis; this area represents a decrease in velocity.

Velocity-Time Curve. Since the acceleration is either constant or zero, the v–t curve is made of straight-line segments connecting the points determined above.

\[\text{Change in } x = \text{area under } v–t \text{ curve}\]
\[0 < t < 6: \quad x₆ - 0 = \frac{1}{2}(6)(24) = 72 \text{ m}\]

\[6 < t < 8: \quad x_{10} - x₆ = \frac{1}{2}(4)(24 + 36) = 120 \text{ m}\]

\[8 < t < 34: \quad x_{34} - x_{10} = (42)(36) = 1512 \text{ m}\]

\[34 < t < 40: \quad x_{40} - x_{34} = \frac{1}{2}(6)(36) = 108 \text{ m}\]
PROBLEM 11.61 (Continued)

Adding the changes in \( x \), we obtain the distance from \( A \) to \( B \):

\[ d = x_{40} - 0 = 1812 \text{ m} \]

\[ d = 1812 \text{ m} \]

**Position-Time Curve.** The points determined above should be joined by three arcs of parabola and one straight-line segment.
**PROBLEM 11.62**

For the particle and motion of Problem 11.61, plot the $v-t$ and $x-t$ curves for $0 < t < 20$ s and determine (a) the maximum value of the velocity of the particle, (b) the maximum value of its position coordinate.

**SOLUTION**

(a)

Initial conditions: $t = 0, \quad v_0 = -6 \, \text{m/s}, \quad x_0 = 0$

Change in $v$ equals area under $a-t$ curve:

0 < $t$ < 4 s: $v_4 - v_0 = (1 \, \text{m/s}^2)(4 \, \text{s}) = 4 \, \text{m/s}$

4 s < $t$ < 10 s: $v_{10} - v_4 = (2 \, \text{m/s}^2)(6 \, \text{s}) = +12 \, \text{m/s}$

10 s < $t$ < 12 s: $v_{12} - v_{10} = (-2 \, \text{m/s}^2)(2 \, \text{s}) = -4 \, \text{m/s}$

12 s < $t$ < 20 s: $v_{20} - v_{12} = (-2 \, \text{m/s}^2)(8 \, \text{s}) = -16 \, \text{m/s}$

Change in $x$ equals area under $v-t$ curve:

0 < $t$ < 4 s: $x_4 - x_0 = \frac{1}{2}(-6 - 2)(4) = -16 \, \text{m}$

4 s < $t$ < 5 s: $x_5 - x_4 = \frac{1}{2}(-2)(1) = -1 \, \text{m}$

5 s < $t$ < 10 s: $x_{10} - x_5 = \frac{1}{2}(+10)(5) = +25 \, \text{m}$

10 s < $t$ < 12 s: $x_{12} - x_{10} = \frac{1}{2}(+10 + 6)(2) = +16 \, \text{m}$
PROBLEM 11.62 (Continued)

12 s < t < 15 s:
\[ x_{16} - x_{12} = \frac{1}{2} (+6)(3) = +9 \text{ m} \]
\[ x_{16} = +33 \text{ m} \]

15 s < t < 20 s:
\[ x_{20} - x_{15} = \frac{1}{2} (-10)(5) = -25 \text{ m} \]
\[ x_{20} = +8 \text{ m} \]

From \( v-t \) and \( x-t \) curves, we read

(a) At \( t = 10 \text{ s} \):
\[ v_{\text{max}} = +10.00 \text{ m/s} \]

(b) At \( t = 15 \text{ s} \):
\[ x_{\text{max}} = +33.0 \text{ m} \]
PROBLEM 11.63

A particle moves in a straight line with the velocity shown in the figure. Knowing that $x = -180$ m at $t = 0$,

(a) construct the $a-t$ and $x-t$ curves for $0 < t < 50$ s, and determine (b) the total distance traveled by
the particle when $t = 50$ s, (c) the two times at which $x = 0$.

**SOLUTION**

(a) $a = \text{slope of } v-t$ curve at time $t$

From $t = 0$ to $t = 10$ s:
$v = \text{constant} \Rightarrow a = 0$

$t = 10$ s to $t = 26$ s:
$a = \frac{-10 - 20}{26 - 10} = -\frac{30}{16} = -1.875 \text{ m/s}^2$

$t = 26$ s to $t = 41$ s:
$v = \text{constant} \Rightarrow a = 0$

$t = 41$ s to $t = 46$ s:
$a = \frac{-4 - (-10)}{46 - 41} = 1.2 \text{ m/s}^2$

$t > 46$ s:
$v = \text{constant} \Rightarrow a = 0$

$x_2 = x_1 + \text{(area under } v-t \text{ curve from } t_1 \text{ to } t_2)$

At $t = 10$ s:
$x_{10} = -180 + 10(20) = +20$ m

Next, find time at which $v = 0$. Using similar triangles

\[
\frac{t_{v=0} - 10}{20} = \frac{26 - 10}{30} \quad \text{or} \quad t_{v=0} = \frac{62}{3} \text{ s}
\]

At $t = \frac{62}{3}$ s:
$x_{\frac{62}{3}} = 20 + \frac{1}{2} \left( \frac{62}{3} - 10 \right)(20) = \frac{380}{3} \text{ m}$

$t = 26$ s:
$x_{26} = \frac{380}{3} - \frac{1}{2} \left( \frac{26 - 62}{3} \right)(10) = 124 \text{ m}$

$t = 41$ s:
$x_{41} = -124 - 15(10) = -26 \text{ m}$

$t = 46$ s:
$x_{46} = -26 - 5 \left( \frac{10 + 4}{2} \right) = -61 \text{ m}$

$t = 50$ s:
$x_{50} = -61 - 4(4) = -77 \text{ m}$
PROBLEM 11.63 (Continued)

(b) From $t = 0$ to $t = \frac{62}{3}$ s: Distance traveled $= \frac{380}{3} - (-180) = \frac{920}{3}$ m

$t = 22$ s to $t = 50$ s: Distance traveled $= \left| -77 - \frac{380}{3} \right| = \frac{611}{3}$ m

Total distance traveled $= \left( \frac{920}{3} + \frac{611}{3} \right) m = \frac{1531}{3}$ m $= 510.33$ m

(c) Using similar triangles

Between 0 and 10 s: $\frac{(t_{x=0})_1 - 0}{180} = \frac{10}{200}$

$(t_{x=0})_1 = 9.00$ s

Between 26 s and 41 s: constant velocity

$124 \text{ m} = 10(\Delta t)$

$\Delta t = 12.4$

$t = (26 + 12.4) s = 38.4$ s

$(t_{x=0})_2 = 38.4$ s
**PROBLEM 11.64**

A particle moves in a straight line with the velocity shown in the figure. Knowing that $x = -180$ m at $t = 0$, (a) construct the $a - t$ and $x - t$ curves for $0 < t < 50$ s, and determine (b) the maximum value of the position coordinate of the particle, (c) the values of $t$ for which the particle is at $x = 30$ m.

**SOLUTION**

(a) $a = \text{slope of } v - t \text{ curve at time } t$

From $t = 0$ to $t = 10$ s:

$v = \text{constant} \Rightarrow a = 0$

$t = 10$ s to $t = 26$ s: $a = \frac{-10 - 20}{26 - 10} = \frac{-30}{16} = -1.875 \text{ m/s}^2$

$t = 26$ s to $t = 41$ s: $v = \text{constant} \Rightarrow a = 0$

$t = 41$ s to $t = 46$ s: $a = \frac{-4 - (-10)}{46 - 41} = 1.2 \text{ m/s}^2$

$t > 46$ s: $v = \text{constant} \Rightarrow a = 0$

\[ a, m/s^2 \]

\[ t, s \]

\[ -1.875 \]

\[ 10 \]

\[ 26 \]

\[ 12 \]

\[ 50 \]

\[ v = \text{constant} \Rightarrow a = 0 \]

\[ t = 10 \text{ s} \]

\[ x = \frac{-180 + 10(20)}{2} = +20 \text{ m} \]

At $t = 10$ s:

Next, find time at which $v = 0$. Using similar triangles

\[ \frac{t_v = 0 - 10}{20} = \frac{26 - 10}{30} \text{ or } \frac{t_v = 0}{20} = \frac{62}{3} \text{ s} \]

At $t = \frac{62}{3}$ s: $x_{t = 62/3} = 20 + \frac{1}{2} \left( \frac{62}{3} - 10 \right)(20) = \frac{380}{3} \text{ m}$

$t = 26$ s: $x_{t = 26} = \frac{380}{3} - \frac{1}{2} \left( \frac{26 - 62}{3} \right)(10) = 124 \text{ m}$

$t = 41$ s: $x_{t = 41} = 124 - 15(10) = -26 \text{ m}$

$t = 46$ s: $x_{t = 46} = -26 - 5 \left( \frac{10 + 4}{2} \right) = -61 \text{ m}$

$t = 50$ s: $x_{t = 50} = -61 - 4(4) = -77 \text{ m}$
PROBLEM 11.64 (Continued)

(b) Reading from the $x-t$ curve:
\[
\frac{380}{3} \text{ m} = 126.667 \text{ m}
\]

(c) Between 10 s and 62/3 s
\[
30 \text{ m} = \frac{380}{3} \text{ m} - (\text{area under } v-t \text{ curve from } t_1 \text{ to } \frac{62}{3} \text{ s}) \text{ m}
\]

or
\[
30 = \frac{380}{3} - \frac{1}{2} \left( \frac{62}{3} - t_1 \right) (v_1)
\]

or
\[
\left( \frac{62}{3} - t_1 \right) (v_1) = \frac{580}{3}
\]

Using similar triangles
\[
\frac{v_1}{\left( \frac{62}{3} - t_1 \right)} = \frac{20}{32/3} \quad \text{or} \quad v_1 = \frac{15}{8} \left( \frac{62}{3} - t_1 \right)
\]

Then
\[
\left( \frac{62}{3} - t_1 \right) \left[ \frac{15}{8} \left( \frac{62}{3} - t_1 \right) \right] = \frac{580}{3}
\]

or
\[
t_1 = 10.512 \text{ s} \quad \text{and} \quad t_1 = 30.82 \text{ s}
\]

We have
\[
10 \text{ s} < t_1 < 22 \text{ s} \Rightarrow \quad t_1 = 10.52 \text{ s}
\]

Between 26 s and 41 s:

Using similar triangles
\[
\frac{41 - t_2}{56} = \frac{15}{150}
\]

or
\[
t_2 = 35.4 \text{ s}
\]
PROBLEM 11.65

A parachutist is in free fall at a rate of 200 km/h when he opens his parachute at an altitude of 600 m. Following a rapid and constant deceleration, he then descends at a constant rate of 50 km/h from 586 m to 30 m, where he maneuvers the parachute into the wind to further slow his descent. Knowing that the parachutist lands with a negligible downward velocity, determine (a) the time required for the parachutist to land after opening his parachute, (b) the initial deceleration.

SOLUTION

Assume second deceleration is constant. Also, note that

\[ 200 \text{ km/h} = 55.555 \text{ m/s}, \]
\[ 50 \text{ km/h} = 13.888 \text{ m/s}. \]

(a) Now \( \Delta x = \text{area under } v-t \text{ curve for given time interval} \)

Then \( (586 - 600) \text{ m} = -t_1 \left( \frac{55.555 + 13.888}{2} \right) \text{ m/s} \)

or \( t_1 = 0.4032 \text{ s} \)

(30 - 586) m = \(-t_2 (13.888 \text{ m/s}) \)

or \( t_2 = 40.0346 \text{ s} \)

(0 - 30) m = \(-\frac{1}{2} (t_3) (13.888 \text{ m/s}) \)

or \( t_3 = 4.3203 \text{ s} \)

\( t_{\text{total}} = (0.4032 + 40.0346 + 4.3203) \text{ s} \)

or \( t_{\text{total}} = 44.8 \text{ s} \)

(b) We have \( a_{\text{initial}} = \frac{\Delta v_{\text{initial}}}{t_1} \)

\[ = \frac{[-13.888 - (-55.555)] \text{ m/s}}{0.4032 \text{ s}} \]

\[ = 103.3 \text{ m/s}^2 \]

or \( a_{\text{initial}} = 103.3 \text{ m/s}^2 \)
PROBLEM 11.66

A machine component is spray-painted while it is mounted on a pallet that travels 4 m in 20 s. The pallet has an initial velocity of 80 mm/s and can be accelerated at a maximum rate of 60 mm/s². Knowing that the painting process requires 15 s to complete and is performed as the pallet moves with a constant speed, determine the smallest possible value of the maximum speed of the pallet.

SOLUTION

First note that (80 mm/s)(20 s) < 4000 mm, so that the speed of the pallet must be increased. Since \( v_{\text{paint}} = \text{constant} \), it follows that \( v_{\text{paint}} = v_{\text{max}} \) and then \( t_1 \leq 0.5 \text{s} \). From the \( v-t \) curve, \( A_1 + A_2 = 4000 \text{ mm} \) and it is seen that \( (v_{\text{max}})_\text{min} \) occurs when

\[
a_1 = \frac{v_{\text{max}} - 80}{t_1}
\]

is maximum.

Thus,

\[
\frac{(v_{\text{max}} - 80)\text{mm/s}}{t_1(s)} = 60 \text{ mm/s}
\]

or

\[
t_1 = \frac{(v_{\text{max}} - 80)}{60}
\]

and

\[
t_1\left(\frac{80 + v_{\text{max}}}{2}\right) + (20 - t_1)(v_{\text{max}}) = 4000
\]

Substituting for \( t_1 \)

\[
\frac{(v_{\text{max}} - 80)}{60}\left(\frac{80 + v_{\text{max}}}{2}\right) + \left(20 - \frac{v_{\text{max}} - 80}{60}\right)v_{\text{max}} = 4000
\]

Simplifying

\[
v_{\text{max}}^2 - 2560v_{\text{max}} + 486400 = 0
\]

Solving

\[
v_{\text{max}} = 207 \text{ mm/s} \quad \text{and} \quad v_{\text{max}} = 2353 \text{ mm/s}
\]

For

\[
v_{\text{max}} = 207 \text{ mm/s}, \quad t_1 < 5 \text{ s}
\]

\[
v_{\text{max}} = 2353 \text{ mm/s}, \quad t_1 > 5 \text{ s}
\]

\[
(v_{\text{max}})_\text{min} = 207 \text{ mm/s}
\]
PROBLEM 11.67

A temperature sensor is attached to slider AB, which moves back and forth through 1500 mm. The maximum velocities of the slider are 300 mm/s to the right and 750 mm/s to the left. When the slider is moving to the right, it accelerates and decelerates at a constant rate of 150 mm/s\(^2\); when moving to the left, the slider accelerates and decelerates at a constant rate of 500 mm/s\(^2\). Determine the time required for the slider to complete a full cycle, and construct the \(v-t\) and \(x-t\) curves of its motion.

SOLUTION

The \(v-t\) curve is first drawn as shown. Then

\[
t_a = \frac{v_{\text{right}}}{a_{\text{right}}} = \frac{300 \text{ mm/s}}{150 \text{ mm/s}^2} = 2 \text{ s}
\]

\[
t_d = \frac{v_{\text{left}}}{a_{\text{left}}} = \frac{750 \text{ mm/s}}{500 \text{ mm/s}^2} = 1.5 \text{ s}
\]

Now

\[A_1 = 1500 \text{ mm}\]

or

\[[(t_1 - 2)\ a(300 \text{ mm/s})] = 1500 \text{ mm}\]

or

\[t_1 = 7 \text{ s}\]

and

\[A_2 = 1500 \text{ mm}\]

or

\[\{[(t_2 - 7) - 1.5]s\} (750 \text{ mm/s}) = 1500 \text{ mm}\]

or

\[t_2 = 10.5 \text{ s}\]

Now

\[t_{\text{cycle}} = t_2\]

We have \(x_i = x + (\text{area under \(v-t\) curve from} t_i \text{ to} t_{ij})\)

At \(t = 2 \text{ s}\): \(x_2 = \frac{1}{2} (2)(300) = 300 \text{ mm}\)

\(t = 5 \text{ s}\): \(x_5 = 300 + (5 - 2)(300) = 1200 \text{ mm}\)

\(t = 7 \text{ s}\): \(x_7 = 1500 \text{ mm}\)

\(t = 8.5 \text{ s}\): \(x_{8.5} = 1500 - \frac{1}{2} (1.5)(750) = 937.5 \text{ mm}\)

\(t = 9 \text{ s}\): \(x_9 = 937.5 \text{ mm} - (0.5)(750) = 562.5 \text{ mm}\)

\(t = 10.5 \text{ s}\): \(x_{10.5} = 0\)
**PROBLEM 11.68**

A commuter train traveling at 60 km/h is 4.5 km from a station. The train then decelerates so that its speed is 30 km/h when it is 0.75 km from the station. Knowing that the train arrives at the station 7.5 min after beginning to decelerate and assuming constant decelerations, determine (a) the time required for the train to travel the first 3.75 km, (b) the speed of the train as it arrives at the station, (c) the final constant deceleration of the train.

**SOLUTION**

Given: At \( t = 0 \), \( v = 60 \) km/h, \( x = 0 \); when \( x = 3.75 \) km, \( v = 30 \) km/h; at \( t = 7.5 \) min, \( x = 4.5 \) km; constant decelerations

The \( v-t \) curve is first drawn as shown.

(a) We have

\[
A_1 = 3.75 \text{ km}
\]

or

\[
(t_1 \text{ min}) \left( \frac{60 + 30}{2} \right) \text{ km/h} \times \frac{1 \text{ h}}{60 \text{ min}} = 3.75 \text{ km}
\]

or

\[
t_1 = 5 \text{ min} \quad \downarrow
\]

(b) We have

\[
A_2 = 0.75 \text{ km}
\]

or

\[
(7.5 - 5) \text{ min} \times \left( \frac{30 + v_2}{2} \right) \text{ km/h} \times \frac{1 \text{ h}}{60 \text{ min}} = 0.75 \text{ km}
\]

or

\[
v_2 = 6 \text{ km/h} \quad \downarrow
\]

(c) We have

\[
a_{\text{final}} = a_{12}
\]

\[
= \frac{(6 - 30) \text{ km/h}}{(7.5 - 5) \text{ min}} \times \frac{1000 \text{ m}}{\text{ km}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{1 \text{ h}}{3600 \text{ s}}
\]

or

\[
a_{\text{final}} = -0.0444 \text{ m/s}^2 \quad \downarrow
\]
**PROBLEM 11.69**

Two road rally checkpoints A and B are located on the same highway and are 12 km apart. The speed limits for the first 8 km and the last 4 km of the section of highway are 100 km/h and 70 km/h, respectively. Drivers must stop at each checkpoint, and the specified time between Points A and B is 8 min 20 s. Knowing that a driver accelerates and decelerates at the same constant rate, determine the magnitude of her acceleration if she travels at the speed limit as much as possible.

**SOLUTION**

Given:

- \((v_{\text{max}})_{AC} = 100 \text{ km/h}\)
- \((v_{\text{max}})_{CB} = 70 \text{ km/h}\)
- \(v_A = v_B = 0\)
- \(t_{AB} = 8 \text{ min} 20 \text{ s} = 8 \frac{5}{36} \text{ h}\)
- \(|a| = \text{constant}; \quad v = v_{\text{max}}\)

The \(v-t\) curve is first drawn as shown, where the magnitudes of the slopes (accelerations) of the three inclined lines are equal.

Note: \(8 \text{ min} 20 \text{ s} = \frac{5}{36} \text{ h}\)

At \(t = t_a\) \(x = 8 \text{ km}\)

\(\frac{5}{36} \text{ h}, \quad x = 12 \text{ km}\)

Denoting the magnitude of the accelerations by \(a\), we have

\[
\begin{align*}
    a &= \frac{100}{t_a} \\
    a &= \frac{30}{t_b} \\
    a &= \frac{70}{t_c}
\end{align*}
\]

where \(a\) is in \(\text{km/h}^2\) and the times are in h.

Now \(A_1 = 8 \text{ km}\):

\[
(t_a)(100) - \frac{1}{2}(t_a)(100) - \frac{1}{2}(t_b)(30) = 8
\]
PROBLEM 11.69 (Continued)

Substituting \[ 100t_3 - \frac{1}{2} \left( \frac{100}{a} \right)(100) - \frac{1}{2} \left( \frac{30}{a} \right)(30) = 8 \]

or \[ t_3 = 0.08 + \frac{54.5}{a} \]

Also \[ A_2 = 4 \text{ km:} \]

\[ \left( \frac{5}{36} - t_1 \right)(70) - \frac{1}{2}(t_1)(70) = 4 \]

Substituting \[ \left( \frac{5}{36} - t_1 \right)(70) - \frac{1}{2}(70) = 4 \]

or \[ t_1 = \frac{103}{1260} - \frac{35}{a} \]

Then \[ 0.08 = \frac{54.5}{a} = \frac{103}{1260} - \frac{35}{a} \]

or \[ a = 51.259 \text{ km/h}^2 \times \frac{1000 \text{ m}}{\text{km}} \times \left( \frac{1 \text{ h}}{3600 \text{ s}} \right)^2 \]

or \[ a = 3.96 \text{ m/s}^2 \]
**PROBLEM 11.70**

In a water-tank test involving the launching of a small model boat, the model’s initial horizontal velocity is 6 m/s, and its horizontal acceleration varies linearly from \(-12\) m/s\(^2\) at \(t = 0\) to \(-2\) m/s\(^2\) at \(t = t_1\) and then remains equal to \(-2\) m/s\(^2\) until \(t = 1.4\) s. Knowing that \(v = 1.8\) m/s when \(t = t_1\), determine (a) the value of \(t_1\), (b) the velocity and the position of the model at \(t = 1.4\) s.

**SOLUTION**

Given:

\[ v_0 = 6 \text{ m/s}; \quad \text{for} \quad 0 \leq t \leq t_1, \quad a \times t; \]

for \(t_1 \leq t \leq 1.4\) s \(a = -2\) m/s\(^2\); at \(t = 0\) \(a = -12\) m/s\(^2\); at \(t = t_1\) \(a = -2\) m/s\(^2\), \(v = 1.8\) m/s\(^2\)

The \(a-t\) and \(v-t\) curves are first drawn as shown.

(a) We have \(v = v_0 + A_1\)

or \(1.8\) m/s = \(6\) m/s – \((t_1 \times \) m/s\(^2\))

or \(t_1 = 0.6\) s

(b) We have \(v = v_0 + A_2\)

or \(v = 1.8\) m/s – \((1.4 - 0.6) \times 2\) m/s\(^2\)

or \(v = 0.20\) m/s

Now \(x = A_3 + A_4\), where \(A_3\) is most easily determined using integrating.

Thus, for \(0 \leq t \leq t_1\):

\[ a = \frac{-2 - (-12)}{0.6} t - 12 = \frac{50}{3} t - 12 \]

Now

\[ \frac{dv}{dt} = a \]

\[ = \frac{50}{3} t - 12 \]
PROBLEM 11.70 (Continued)

At \( t = 0, v = 6 \text{ m/s} \):

\[
\int_0^v dv = \int_0^t \left( \frac{50}{3} t - 12 \right) dt
\]

or

\[ v = 6 + \frac{25}{3} t^2 - 12 t \]

We have

\[
\frac{dx}{dt} = v = 6 - 12t + \frac{25}{3} t^2
\]

Then

\[
A_3 = \int_0^t dx = \int_0^0 \left( 6 - 12t + \frac{25}{3} t^2 \right) dt
\]

\[
= 6t - 6t^2 + \frac{25}{9} t^3 \bigg|_{t=0}^{t=0.6} = 2.04 \text{ m}
\]

Also

\[
A_4 = (1.4 - 0.6) \left( \frac{1.8 + 0.2}{2} \right) = 0.8 \text{ m}
\]

Then

\[ x_{1.4} = (2.4 + 0.8) \text{ m} \]

or

\[ x_{1.4} = 2.84 \text{ m} \]

PROBLEM 11.71

A car and a truck are both traveling at the constant speed of 50 km/h; the car is 12 m behind the truck. The driver of the car wants to pass the truck, i.e., he wishes to place his car at B, 12 m in front of the truck, and then resume the speed of 50 km/h. The maximum acceleration of the car is 1.5 m/s² and the maximum deceleration obtained by applying the brakes is 6 m/s². What is the shortest time in which the driver of the car can complete the passing operation if he does not at any time exceed a speed of 75 km/h? Draw the v-t curve.

SOLUTION

Relative to truck, car must move a distance:  \[ \Delta x = 4.8 + 12 + 15 + 12 = 43.8 \text{ m} \]

Allowable increase in speed:  \[ \Delta v_m = 75 - 50 = 25 \text{ km/h} = \frac{125}{18} \text{ m/s} \]

**Acceleration Phase:**
\[
t_1 = \frac{125/18}{1.5} = \frac{125}{27} \text{ s} = 4.6296 \text{ s}
\]

\[
A_1 = \frac{1}{2} \left( \frac{125}{18} \right) \left( \frac{125}{27} \right) = 16.075 \text{ m}
\]

**Deceleration Phase:**
\[
t_3 = \frac{125/18}{6} = \frac{125}{108} \text{ s} = 1.1574 \text{ s}
\]

\[
A_3 = \frac{1}{2} \left( \frac{125}{18} \right) \left( \frac{125}{100} \right) = 4.0188 \text{ m}
\]

But:  \[ \Delta x = A_1 + A_2 + A_3; \]
\[
43.8 = 16.075 + (125/18)t_2 + 4.0188
\]

\[
t_2 = 3.414 \text{ s}
\]

\[
t_{total} = t_1 + t_2 + t_3 = 4.6296 \text{ s} + 3.414 \text{ s} + 1.15745 \text{ s} = 9.201 \text{ s}
\]

\[
t_F = 9.20 \text{ s} \uparrow
\]
PROBLEM 11.72

Solve Problem 11.71, assuming that the driver of the car does not pay any attention to the speed limit while passing and concentrates on reaching position B and resuming a speed of 50 km/h in the shortest possible time. What is the maximum speed reached? Draw the $v-t$ curve.

SOLUTION

Relative to truck, car must move a distance:

$$\Delta x = 4.8 + 12 + 15 + 12 = 43.8 \text{ m}$$

$$\Delta x = A_1 + A_2 :$$

$$43.8 \text{ m} = \frac{1}{2} (\Delta v_m)(t_1 + t_2)$$

$$43.8 \text{ m} = \frac{1}{2} (1.5t_1)(t_1 + \frac{t_1}{4})$$

$$t_1^2 = 46.72 \quad t_1 = 6.835 \text{ s} \quad t_2 = \frac{1}{4}t_1 = 1.709$$

$$t_{total} = t_1 + t_2 = 6.835 + 1.709 \quad t_f = 8.54 \text{ s}$$

$$\Delta v_m = 1.5t_1 = 1.5(6.835) = 10.2525 \text{ m/s} = 36.909 \text{ m/s}$$

Speed $v_{truck} = 50 \text{ km/h}$, $v_m = 50 \text{ km/h} + 36.91 \text{ km/h}$

$v_m = 86.9 \text{ km/h}$
PROBLEM 11.73

An elevator starts from rest and moves upward, accelerating at a rate of 1.2 m/s² until it reaches a speed of 7.8 m/s, which it then maintains. Two seconds after the elevator begins to move, a man standing 12 m above the initial position of the top of the elevator throws a ball upward with an initial velocity of 20 m/s. Determine when the ball will hit the elevator.

SOLUTION

Given:
At \( t = 0 \), \( v_E = 0 \); For \( 0 \leq v_E < 7.8 \) m/s,
\( a_E = 1.2 \) m/s²; For \( v_E = 7.8 \) m/s, \( a_E = 0 \);
At \( t = 2 \) s, \( v_B = 20 \) m/s

The \( v-t \) curves of the ball and the elevator are first drawn as shown. Note that the initial slope of the curve for the elevator is \( 1.2 \) m/s², while the slope of the curve for the ball is \( -g \) (\( -9.81 \) m/s²).

The time \( t_1 \) is the time when \( v_E \) reaches 7.8 m/s.
Thus, \( v_E = (0) + a_E t \)
or \( 7.8 \) m/s = \( (1.2 \) m/s²\) \( t_1 \)
or \( t_1 = 6.5 \) s

The time \( t_{top} \) is the time at which the ball reaches the top of its trajectory.
Thus, \( v_B = (v_B)_0 - g(t - 2) \)
or \( 0 = 20 \) m/s - \( (9.81 \) m/s²\) \( (t_{top} - 2) \) s
or \( t_{top} = 4.0387 \) s
PROBLEM 11.73 (Continued)

Using the coordinate system shown, we have

\[ 0 \leq t \leq t_1: \]
\[ y_E = -12 \, \text{m} + \left( \frac{1}{2} a t^2 \right) \, \text{m} \]

At \( t = t_{top} \):
\[ y_B = \frac{1}{2} (4.0387 - 2) \, \text{s} \times (20 \, \text{m/s}) \]
\[ = 20.387 \, \text{m} \]

and
\[ y_E = -12 \, \text{m} + \frac{1}{2} (1.2 \, \text{m/s}^2)(4.0387 \, \text{s})^2 \]
\[ = -2.213 \, \text{m} \]

At \( t = t_{top} \),
\[ t = [2 + 2(4.0387 - 2)] \, \text{s} = 6.0774 \, \text{s}, \quad y_B = 0 \]

and at \( t = t_1 \),
\[ y_E = -12 \, \text{m} + \frac{1}{2} (6.5 \, \text{s})(7.8 \, \text{m/s}) = 13.35 \, \text{m} \]

The ball hits the elevator \((y_B = y_E)\) when \( t_{top} < t < t_1 \).

For \( t \geq t_{top} \):
\[ y_B = 20.387 \, \text{m} - \left[ \frac{1}{2} g (t - t_{top})^2 \right] \, \text{m} \]

Then,
when \( y_B = y_E \)
\[ 20.387 \, \text{m} - \frac{1}{2} (9.81 \, \text{m/s}^2)(t - 4.0387)^2 \]
\[ = -12 \, \text{m} + \frac{1}{2} (1.2 \, \text{m/s}^2)(t \, \text{s})^2 \]

or
\[ 5.505t^2 - 39.6196t + 47.619 = 0 \]

Solving
\[ t = 1.525 \, \text{s} \quad \text{and} \quad t = 5.67 \, \text{s} \]

Choosing the smaller value
\[ t = 1.525 \, \text{s} \]
PROBLEM 11.74

The acceleration record shown was obtained for a small airplane traveling along a straight course. Knowing that \( x(0) = 0 \) and \( v(0) = 60 \, \text{m/s} \) when \( t = 0 \), determine (a) the velocity and position of the plane at \( t = 20 \, \text{s} \), (b) its average velocity during the interval \( 6 \, \text{s} < t < 14 \, \text{s} \).

SOLUTION

Geometry of “bell-shaped” portion of \( v-t \) curve

The parabolic spandrels marked by * are of equal area. Thus, total area of shaded portion of \( v-t \) diagram is:

\[
\Delta x = 6 \, \text{m}
\]

(a) When \( t = 20 \, \text{s} \):

\[
x_{20} = (60 \, \text{m/s})(20 \, \text{s}) \quad \text{(shaded area)}
\]

\[
= 1200 \, \text{m} - 6 \, \text{m}
\]

\[
x_{20} = 1194 \, \text{m}
\]

(b) From \( t = 6 \, \text{s} \) to \( t = 14 \, \text{s} \):

\[
\Delta t = 8 \, \text{s}
\]

\[
\Delta x = (60 \, \text{m/s})(14 \, \text{s} - 6 \, \text{s}) \quad \text{(shaded area)}
\]

\[
= (60 \, \text{m/s})(8 \, \text{s}) - 6 \, \text{m} = 480 \, \text{m} - 6 \, \text{m} = 474 \, \text{m}
\]

\[
v_{\text{average}} = \frac{\Delta x}{\Delta t} = \frac{474 \, \text{m}}{8 \, \text{s}} \quad \text{v_{average} = 59.25 \, \text{m/s}}
\]
PROBLEM 11.75

Car A is traveling on a highway at a constant speed \((v_A)_0 = 90 \text{ km/h}\) and is 120 m from the entrance of an access ramp when car B enters the acceleration lane at that point at a speed \((v_B)_0 = 25 \text{ km/h}\). Car B accelerates uniformly and enters the main traffic lane after traveling 60 m in 5 s. It then continues to accelerate at the same rate until it reaches a speed of 90 km/h, which it then maintains. Determine the final distance between the two cars.

SOLUTION

Given:

\[
\begin{align*}
(v_A)_0 &= 90 \text{ km/h}, \quad (v_B)_0 = 25 \text{ km/h}; \quad \text{at } t = 0, \\
(x_A)_0 &= -120 \text{ m}, \quad (x_B)_0 = 0; \quad \text{at } t = 5 \text{ s}, \\
x_B &= 60 \text{ m}; \quad \text{for } 25 \text{ km/h} \leq v_B < 90 \text{ km/h}, \\
\text{a}_B &= \text{constant}; \quad \text{for } v_B = 90 \text{ km/h}, \\
\text{a}_B &= 0
\end{align*}
\]

First note

\[
\begin{align*}
90 \text{ km/h} &= 25 \text{ m/s} \\
25 \text{ km/h} &= 125/18 \text{ m/s}
\end{align*}
\]

The \(v-t\) curves of the two cars are then drawn as shown.

Using the coordinate system shown, we have

\[
\begin{align*}
at t = 5 \text{ s}, \quad x_B &= 60 \text{ m}: \quad (5 \text{ s}) \left[ \frac{(125/18 + v_B)_5}{2} \right] \text{m/s} = 60 \text{ m} \\
or \quad (v_B)_5 &= \frac{307}{18} \text{ m/s} = 17.0556 \text{ m/s}
\end{align*}
\]

Then, using similar triangles, we have

\[
\frac{(25 - 125/18) \text{ m/s}}{t_1} = \frac{(307/18 - 125/18) \text{ m/s}}{5 \text{ s}} (= \text{a}_B)
\]

or

\[
t_1 = 8.9286 \text{ s}
\]

Finally, at \(t = t_1\)

\[
\begin{align*}
x_{B/A} &= x_B - x_A = \left[ (8.9286 \text{ s}) \left( \frac{25 + 125/18}{2} \right) \text{m/s} \right] \\
&\quad - [-120 \text{ m} + (8.9286 \text{ s})(25 \text{ m/s})] \\
or \\
x_{B/A} &= 39.4 \text{ m}
\end{align*}
\]
PROBLEM 11.76

Car A is traveling at 60 km/h when it enters a 40 km/h speed zone. The driver of car A decelerates at a rate of 5 m/s² until reaching a speed of 40 km/h, which she then maintains. When car B, which was initially 20 m behind car A and traveling at a constant speed of 70 km/h, enters the speed zone, its driver decelerates at a rate of 6 m/s² until reaching a speed of 35 km/h. Knowing that the driver of car B maintains a speed of 35 km/h, determine (a) the closest that car B comes to car A, (b) the time at which car A is 25 m in front of car B.

SOLUTION

Given:

\( (v_A)_0 = 60 \text{ km/h}; \) For 40 km/h < \( v_A \leq 60 \text{ km/h}; \)

\( a_A = -5 \text{ m/s}^2; \) For \( v_A = 40 \text{ km/h}, a_A = 0; \)

\( (x_{AB})_0 = 20 \text{ m}; \) \( (v_B)_0 = 70 \text{ km/h}; \)

when \( x_B = 0, \) \( a_B = -6 \text{ m/s}^2; \) for \( v_B = 35 \text{ km/h}, \)

\( a_B = 0. \)

First note:

\( 60 \text{ km/h} = \frac{50}{3} \text{ m/s}, \)

\( 40 \text{ km/h} = \frac{100}{9} \text{ m/s} \)

\( 70 \text{ km/h} = \frac{175}{9} \text{ m/s}, \)

\( 35 \text{ km/h} = \frac{175}{18} \text{ m/s} \)

At \( t = 0 \)

The \( v-t \) curves of the two cars are as shown.

At \( t = 0: \) car A enters the speed zone

\( t = (t_B)_1: \) car B enters the speed zone

\( t = t_A: \) car A reaches its final speed

\( t = t_{min}: \) \( v_A = v_B \)

\( t = (t_B)_2: \) car B reaches its final speed
PROBLEM 11.76 (Continued)

(a) We have
\[ a_A = \frac{(v_A)_{\text{final}} - (v_A)_0}{t_A} \]
or
\[ -5 \text{ m/s}^2 = \frac{(100 - 50)}{3} \text{ m/s} \]
or
\[ t_A = 1.1111 \text{s} \]
Also
\[ 20 \text{ m} = (t_B)_1(v_B)_0 \]
or
\[ 20 \text{ m} = (t_B)_1\left(\frac{175}{9} \text{ m/s}\right) \quad \text{or} \quad (t_B)_1 = 1.0286 \text{s} \]
and
\[ a_B = \frac{(v_B)_{\text{final}} - (v_B)_0}{(t_B)_2 - (t_B)_1} \]
or
\[ -6 \text{ m/s}^2 = \frac{(175 - 175)}{18} \text{ m/s} \quad \text{or} \quad (t_B)_2 - (1.0286) \text{s} \]
Car B will continue to overtake car A while \( v_B > v_A \). Therefore, \((x_{A/B})_{\text{min}}\) will occur when \( v_A = v_B \), which occurs for
\[ (t_B)_1 < t_{\text{min}} < (t_B)_2 \]
For this time interval
\[ v_A = \frac{100}{9} \text{ m/s} \]
\[ v_B = (v_B)_0 + a_B(t - (t_B)_1) \]
Then at \( t = t_{\text{min}}^2 \):
\[ \frac{100}{9} = \frac{175}{9} + (-6 \text{ m/s}^2)(t_{\text{min}} - 1.0286) \text{s} \]
or
\[ t_{\text{min}} = 2.4175 \text{s} \]
Finally
\[ (x_{A/B})_{\text{min}} = (x_A)_{t_{\text{min}}} - (x_B)_{t_{\text{min}}} \]
\[ = \{ t_A \left[ \frac{(v_A)_0 + (v_A)_{\text{final}}}{2} \right] + (t_{\text{min}} - t_A)(v_A)_{\text{final}} \} \]
\[ - \{ (x_B)_0 + (t_B)_1(v_B)_0 + [t_{\text{min}} - (t_B)_1] \left[ \frac{(v_B)_0 + (v_B)_{\text{final}}}{2} \right] \} \]
PROBLEM 11.76 (Continued)

\[ = \left[ (1.1111 s) \left( \frac{50 + 100}{3 + 9} \right) \frac{m}{s} + (2.4175 - 1.1111 s) \times \frac{100}{9} \frac{m}{s} \right] \\
- \left[ -20 m + (1.0286 s) \left( \frac{175}{9} \frac{m}{s} \right) + (2.4175 - 1.0286 s) \times \left( \frac{175}{9} + \frac{100}{2} \right) \frac{m}{s} \right] \\
= (15.432 + 14.516) m - (-20 + 20.00 + 21.219) m \\
= 8.729 m \]

or \( (x_{A/B})_{min} = 8.73 m \) \( \blacksquare \)

(b) Since \( (x_{A/B}) \leq 20 m \) for \( t \leq t_{min} \), it follows that \( x_{A/B} = 25 m \) for \( t > (t_B)_2 \)

[Note \( (t_B)_2 = t_{min} \)]. Then, for \( t > (t_B)_2 \)

\[ x_{A/B} = (x_{A/B})_{min} + [(t - t_{min})(v_A)_{final}] \\
- \left\{ [(t_B)_2 - (t_{min})][\frac{(v_A)_{final} + (v_B)_{final}}{2}] + [t - (t_B)_2](v_B)_{final} \right\} \\
\]

or \( 25 m = 8.729 m + \left[ (t - 2.4175) s \times \frac{100}{9} \frac{m}{s} \right] \\
- \left( 2.6489 - 2.4175 \right) s \times \left( \frac{100}{9} + \frac{175}{18} \right) \frac{m}{s} + (t - 2.6489) s \times \left( \frac{175}{18} \right) \frac{m}{s} \)

or \( t = 14.25 s \) \( \blacksquare \)
PROBLEM 11.77

A car is traveling at a constant speed of 54 km/h when its driver sees a child run into the road. The driver applies her brakes until the child returns to the sidewalk and then accelerates to resume her original speed of 54 km/h; the acceleration record of the car is shown in the figure. Assuming x = 0 when t = 0, determine (a) the time $t_1$ at which the velocity is again 54 km/h, (b) the position of the car at that time, (c) the average velocity of the car during the interval $1 \leq t \leq t_1$.

SOLUTION

Given: 
At $t = 0$, $x = 0$, $v = 54$ km/h; for $t = t_1$, $v = 54$ km/h

First note 54 km/h = 15 m/s

(a) We have $v = v_a + \text{(area under a–t curve from } t_a \text{ to } t_b)$

Then at $t = 2$ s: $v = 15 - (1)(6) = 9$ m/s

$t = 4.5$ s: $v = 9 - \frac{1}{2}(2.5)(6) = 1.5$ m/s

$t = t_1$: $15 = 1.5 + \frac{1}{2}(t_1 - 4.5)(2)$

or $t_1 = 18$ s

(b) Using the above values of the velocities, the v–t curve is drawn as shown.
PROBLEM 11.77 (Continued)

Now \( x \) at \( t = 18 \text{ s} \)

\[
x_{18} = 0 + \sum \text{ (area under the } v-t \text{ curve from } t = 0 \text{ to } t = 18 \text{ s)}
\]

\[
= (1 \text{ s})(15 \text{ m/s}) + (1 \text{ s})\left( \frac{15 + 9}{2} \right) \text{ m/s}
\]

\[
+ \left[ (2.5 \text{ s})(1.5 \text{ m/s}) + \frac{1}{3}(2.5 \text{ s})(7.5 \text{ m/s}) \right]
\]

\[
+ \left[ (13.5 \text{ s})(1.5 \text{ m/s}) + \frac{2}{3}(13.5 \text{ s})(13.5 \text{ m/s}) \right]
\]

\[
= [15 + 12 + (3.75 + 6.25) + (20.25 + 121.50)] \text{ m}
\]

\[
= 178.75 \text{ m}
\]

or \( x_{18} = 178.8 \text{ m} \)

(c) First note \( x_i = 15 \text{ m} \)

\( x_{18} = 178.75 \text{ m} \)

Now \( v_{\text{ave}} = \frac{\Delta x}{\Delta t} = \frac{(178.75 - 15) \text{ m}}{(18 - 1) \text{ s}} = 9.6324 \text{ m/s} \)

or \( v_{\text{ave}} = 34.7 \text{ km/h} \)
PROBLEM 11.78

As shown in the figure, from \( t = 0 \) to \( t = 4 \) s, the acceleration of a given particle is represented by a parabola. Knowing that \( x = 0 \) and \( v = 8 \) m/s when \( t = 0 \), (a) construct the \( v-t \) and \( x-t \) curves for \( 0 < t < 4 \) s, (b) determine the position of the particle at \( t = 3 \) s. (Hint: Use table inside the front cover.)

SOLUTION

Given: 
At \( t = 0 \), \( x = 0 \), \( v = 8 \) m/s

(a) We have \( v_2 = v_1 + \text{(area under } a-t \text{ curve from } t_1 \text{ to } t_2) \)

and \( x_2 = x_1 + \text{(area under } v-t \text{ curve from } t_1 \text{ to } t_2) \)

Then, using the formula for the area of a parabolic spandrel, we have

\[
\begin{align*}
\text{at } t = 2 \text{ s: } v &= 8 - \frac{1}{3}(2)(12) = 0 \\
\text{at } t = 4 \text{ s: } v &= 0 - \frac{1}{3}(2)(12) = -8 \text{ m/s}
\end{align*}
\]

The \( v-t \) curve is then drawn as shown.

Now at \( t = 2 \) s: \( x = 0 + \frac{(2)(8)}{3+1} = 4 \) m

\( t = 4 \) s: \( x = 4 - \frac{(2)(8)}{3+1} = 0 \)

The \( x-t \) curve is then drawn as shown.
PROBLEM 11.78 (Continued)

(b) We have

At \( t = 3 \) s:

\[
a = -3(3 - 2)^2 = -3 \text{ m/s}^2
\]

\[
v = 0 - \frac{1}{3}(1)(3) = -1 \text{ m/s}
\]

\[
x = 4 - \frac{(1)(1)}{3+1}
\]

or \( x_3 = 3.75 \text{ m} \)
**PROBLEM 11.79**

During a manufacturing process, a conveyor belt starts from rest and travels a total of 400 mm before temporarily coming to rest. Knowing that the jerk, or rate of change of acceleration, is limited to \( \pm 1.5 \, \text{m/s}^2 \) per second, determine (a) the shortest time required for the belt to move 400 mm, (b) the maximum and average values of the velocity of the belt during that time.

**SOLUTION**

Given:

- At \( t = 0 \), \( x = 0 \), \( v = 0 \);
- \( x_{\text{max}} = 400 \text{ mm} = 0.4 \text{ m} \)

when

\[
\text{at} \quad x = x_{\text{max}}, \quad v = 0; \quad \left| \frac{\text{d}a}{\text{d}t} \right|_{\text{max}} = 1.5 \, \text{m/s}^3
\]

(a) Observing that \( v_{\text{max}} \) must occur at \( t = \frac{1}{2}t_{\text{min}} \), the \( a-t \) curve must have the shape shown. Note that the magnitude of the slope of each portion of the curve is \( 1.5 \, \text{m/s}^2/\text{s} \).

\[
\begin{align*}
\alpha, \text{ m/s}^2
\end{align*}
\]

We have at \( t = \Delta t \):

\[
\begin{align*}
v &= 0 + \frac{1}{2}(\Delta t)(a_{\text{max}}) = \frac{1}{2}a_{\text{max}}\Delta t
\end{align*}
\]

\[
\begin{align*}
t = 2\Delta t : \quad v_{\text{max}} &= \frac{1}{2}a_{\text{max}}\Delta t + \frac{1}{2}(\Delta t)(a_{\text{max}}) = a_{\text{max}}\Delta t
\end{align*}
\]

Using symmetry, the \( v-t \) is then drawn as shown.

\[
\begin{align*}
\end{align*}
\]

Noting that \( A_1 = A_2 = A_3 = A_4 \) and that the area under the \( v-t \) curve is equal to \( x_{\text{max}} \), we have

\[
\begin{align*}
(2\Delta t)(v_{\text{max}}) &= x_{\text{max}}
\end{align*}
\]

\[
\begin{align*}
v_{\text{max}} &= a_{\text{max}}\Delta t \Rightarrow 2a_{\text{max}}\Delta t^2 = x_{\text{max}}
\end{align*}
\]
PROBLEM 11.79 (Continued)

Now \( \frac{a_{\text{max}}}{\Delta t} = 1.5 \text{ m/s}^2 / \text{s} \) so that

\[
2(1.5\Delta t \text{ m/s}^3)\Delta t^2 = 0.4 \text{ m}
\]

or \( \Delta t = 0.5109 \text{ s} \)

Then \( t_{\text{min}} = 4\Delta t \)

or \( t_{\text{min}} = 2.04 \text{ s} \)

(b) We have

\[
v_{\text{max}} = a_{\text{max}}\Delta t = (1.5 \text{ m/s}^2 / \text{s} \times \Delta t)\Delta t = 1.5 \text{ m/s}^2 / \text{s} \times (0.5109)^2
\]

or

\[
v_{\text{max}} = 0.392 \text{ m/s}
\]

Also

\[
v_{\text{ave}} = \frac{\Delta x}{\Delta t_{\text{total}}} = \frac{0.4 \text{ m}}{2.04 \text{ s}}
\]

or

\[
v_{\text{ave}} = 0.196 \text{ m/s}
\]
PROBLEM 11.80

An airport shuttle train travels between two terminals that are 2.5 km apart. To maintain passenger comfort, the acceleration of the train is limited to ±1.2 m/s², and the jerk, or rate of change of acceleration, is limited to ±0.24 m/s³ per second. If the shuttle has a maximum speed of 30 km/h, determine (a) the shortest time for the shuttle to travel between the two terminals, (b) the corresponding average velocity of the shuttle.

SOLUTION

Given:

\[ x_{\text{max}} = 2.5 \text{ km}; \quad |a_{\text{max}}| = 1.2 \text{ m/s}^2 \]
\[ \left| \frac{\text{d}a}{\text{d}t} \right|_{\text{max}} = 0.24 \text{ m/s}^3; \quad v_{\text{max}} = 30 \text{ km/h} \]

First note

\[ 30 \text{ km/h} = \frac{25}{3} \text{ m/s} \]
\[ 2.5 \text{ km} = 2500 \text{ m} \]

(a) To obtain \( t_{\text{min}} \), the train must accelerate and decelerate at the maximum rate to maximize the time for which \( v = v_{\text{max}} \). The time \( \Delta t \) required for the train to have an acceleration of 1.2 m/s² is found from

\[ \frac{\text{d}a}{\text{d}t} = \frac{a_{\text{max}}}{\Delta t} \]

or

\[ \Delta t = \frac{1.2 \text{ m/s}^2}{0.24 \text{ m/s}^3/\text{s}} \]

or

\[ \Delta t = 5 \text{ s} \]

Now,

after 5 s, the speed of the train is

\[ v_5 = \frac{1}{2} (\Delta t)(a_{\text{max}}) \]

\[ \text{since} \quad \frac{\text{d}a}{\text{d}t} = \text{constant} \]

or

\[ v_5 = \frac{1}{2} (5 \text{ s})(1.2 \text{ m/s}^2) = 3 \text{ m/s} \]

Then, since \( v_5 < v_{\text{max}} \), the train will continue to accelerate at 1.2 m/s² until \( v = v_{\text{max}} \). The a–t curve must then have the shape shown. Note that the magnitude of the slope of each inclined portion of the curve is 0.24 m/s³/s.
PROBLEM 11.80 (Continued)

Now at \( t = (10 + \Delta t) \) s, \( v = v_{\text{max}} : \)

\[
2 \left[ \frac{1}{2} (5 \text{ s})(1.2 \text{ m/s}^2) \right] + (\Delta t)(1.2 \text{ m/s}^2) = \frac{25}{3} \text{ m/s}
\]

or \( \Delta t_1 = 1.9444 \) s

Then at \( t = 5 \) s: \( v = 0 + \frac{1}{2}(5)(1.2) = 3 \text{ m/s} \)

\( t = 6.9444 \) s: \( v = 3 + (1.9444)(1.2) = 5.33328 \text{ m/s} \)

\( t = 11.9444 \) s: \( v = 5.33328 + \frac{1}{2}(5)(1.2) = 8.33328 \text{ m/s} \)

Using symmetry, the \( v - t \) curve is then drawn as shown.

Noting that \( A_1 = A_2 = A_3 = A_4 \) and that the area under the \( v - t \) curve is equal to \( x_{\text{max}} \), we have

\[
2 \left[ (1.9444 \text{ s}) \left( \frac{3 + 5.33328}{2} \right) \text{ m/s} \right] + (10 + \Delta t_2) \text{ s} \times (8.33328 \text{ m/s}) = 2500 \text{ m}
\]

or \( \Delta t_2 = 288.06 \) s

Then \( t_{\text{min}} = 4(5 \text{ s}) + 2(1.9444 \text{ s}) + 288.06 \text{ s} \)

\( = 311.95 \) s

or \( t_{\text{min}} = 5 \text{ min}11.95 \) s

(b) We have \( v_{\text{ave}} = \frac{\Delta x}{\Delta t} = \frac{2500 \text{ m}}{311.95 \text{ s}} = 8.014 \text{ m/s} = 28.850 \text{ km/h} \)

or \( v_{\text{ave}} = 28.9 \text{ km/h} \)
PROBLEM 11.81

The acceleration record shown was obtained during the speed trials of a sports car. Knowing that the car starts from rest, determine by approximate means (a) the velocity of the car at \( t = 8 \text{ s} \), (b) the distance the car has traveled at \( t = 20 \text{ s} \).

\[ a (\text{m/s}^2) \]

\[ \begin{array}{|c|c|}
\hline
0 & 0 \\
2 & 1 \\
4 & 2 \\
6 & 3 \\
8 & 4 \\
10 & 5 \\
12 & 6 \\
14 & 7 \\
16 & 6 \\
18 & 5 \\
20 & 4 \\
22 & 3 \\
\hline
\end{array} \]

\[ t (\text{s}) \]

\[ 0 \]

\[ 2 \]

\[ 4 \]

\[ 6 \]

\[ 8 \]

\[ 10 \]

\[ 12 \]

\[ 14 \]

\[ 16 \]

\[ 18 \]

\[ 20 \]

\[ 22 \]

SOLUTION

Given: \( a-t \) curve; at \( t = 0, x = 0, v = 0 \)

1. The \( a-t \) curve is first approximated with a series of rectangles, each of width \( \Delta t = 2 \text{ s} \). The area \((\Delta t)(a_{\text{ave}})\) of each rectangle is approximately equal to the change in velocity \( \Delta v \) for the specified interval of time. Thus,

\[ \Delta v = a_{\text{ave}} \Delta t \]

where the values of \( a_{\text{ave}} \) and \( \Delta v \) are given in columns 1 and 2, respectively, of the following table.

2. Noting that \( v_0 = 0 \) and that

\[ v_2 = v_1 + \Delta v_{12} \]

where \( \Delta v_{12} \) is the change in velocity between times \( t_1 \) and \( t_2 \), the velocity at the end of each 2 s interval can be computed; see column 3 of the table and the \( v-t \) curve.

3. The \( v-t \) curve is next approximated with a series of rectangles, each of width \( \Delta t = 2 \text{ s} \). The area \((\Delta t)(v_{\text{ave}})\) of each rectangle is approximately equal to the change in position \( \Delta x \) for the specified interval of time. Thus,

\[ \Delta x = v_{\text{ave}} \Delta t \]

where \( v_{\text{ave}} \) and \( \Delta x \) are given in columns 4 and 5, respectively, of the table.

4. With \( x_0 = 0 \) and noting that

\[ x_2 = x_1 + \Delta x_{12} \]

where \( \Delta x_{12} \) is the change in position between times \( t_1 \) and \( t_2 \), the position at the end of each 2 s interval can be computed; see column 6 of the table and the \( x-t \) curve.
PROBLEM 11.81 (Continued)

<table>
<thead>
<tr>
<th>t, s</th>
<th>a, m/s²</th>
<th>v, m/s</th>
<th>x, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.43</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>5.08</td>
<td>4.12</td>
<td>11.72</td>
</tr>
<tr>
<td>4</td>
<td>3.38</td>
<td>7.60</td>
<td>24.68</td>
</tr>
<tr>
<td>6</td>
<td>1.80</td>
<td>11.07</td>
<td>48.08</td>
</tr>
<tr>
<td>8</td>
<td>1.76</td>
<td>11.73</td>
<td>68.78</td>
</tr>
<tr>
<td>10</td>
<td>2.04</td>
<td>13.87</td>
<td>99.28</td>
</tr>
<tr>
<td>12</td>
<td>1.64</td>
<td>16.11</td>
<td>131.48</td>
</tr>
<tr>
<td>14</td>
<td>1.25</td>
<td>17.92</td>
<td>163.72</td>
</tr>
<tr>
<td>16</td>
<td>1.14</td>
<td>18.56</td>
<td>196.96</td>
</tr>
<tr>
<td>18</td>
<td>1.03</td>
<td>19.15</td>
<td>220.84</td>
</tr>
<tr>
<td>20</td>
<td>0.91</td>
<td>19.64</td>
<td>245.72</td>
</tr>
<tr>
<td>22</td>
<td>0.80</td>
<td>20.05</td>
<td>271.44</td>
</tr>
</tbody>
</table>

(a) At t = 8 s, \( v = 32.58 \text{ m/s} \) or \( v = 117.3 \text{ km/h} \)

(b) At t = 20 s, \( x = 660 \text{ m} \)
PROBLEM 11.82

Two seconds are required to bring the piston rod of an air cylinder to rest; the acceleration record of the piston rod during the 2 s is as shown. Determine by approximate means (a) the initial velocity of the piston rod, (b) the distance traveled by the piston rod as it is brought to rest.

SOLUTION

Given: a–t curve; at \( t = 25 \), \( v = 0 \)

1. The a–t curve is first approximated with a series of rectangles, each of width \( \Delta t = 0.25 \) s. The area \( (\Delta t)(a_{\text{ave}}) \) of each rectangle is approximately equal to the change in velocity \( \Delta v \) for the specified interval of time. Thus,
   \[
   \Delta v = a_{\text{ave}} \Delta t
   \]
   where the values of \( a_{\text{ave}} \) and \( \Delta v \) are given in columns 1 and 2, respectively, of the following table.

2. Now
   \[
   v(2) = v_0 + \int_0^2 a \, dt = 0
   \]
   and approximating the area \( \int_0^2 a \, dt \) under the a–t curve by \( \Sigma a_{\text{ave}} \Delta t = \Sigma \Delta v \), the initial velocity is then equal to
   \[
   v_0 = -\Sigma \Delta v
   \]
   Finally, using
   \[
   v_2 = v_1 + \Delta v_{12}
   \]
   where \( \Delta v_{12} \) is the change in velocity between times \( t_1 \) and \( t_2 \), the velocity at the end of each 0.25 interval can be computed; see column 3 of the table and the v–t curve.

3. The v–t curve is then approximated with a series of rectangles, each of width 0.25 s. The area \( (\Delta t)(v_{\text{ave}}) \) of each rectangle is approximately equal to the change in position \( \Delta x \) for the specified interval of time. Thus
   \[
   \Delta x = v_{\text{ave}} \Delta t
   \]
   where \( v_{\text{ave}} \) and \( \Delta x \) are given in columns 4 and 5, respectively, of the table.
PROBLEM 11.82 (Continued)

4. With \( x_0 = 0 \) and noting that

\[ x_2 = x_1 + \Delta x_{12} \]

where \( \Delta x_{12} \) is the change in position between times \( t_1 \) and \( t_2 \), the position at the end of each 0.25 s interval can be computed; see column 6 of the table and the \( x-t \) curve.

<table>
<thead>
<tr>
<th>( t, s )</th>
<th>( v, m/s )</th>
<th>( a, m/s^2 )</th>
<th>( x, m )</th>
<th>( \Delta v, m/s )</th>
<th>( \Delta x, m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-4.00</td>
<td>1.914</td>
<td>0</td>
<td>1.512</td>
<td>0.318</td>
</tr>
<tr>
<td>0.25</td>
<td>-1.43</td>
<td>-1.110</td>
<td>1.110</td>
<td>0.871</td>
<td>0.219</td>
</tr>
<tr>
<td>0.50</td>
<td>-1.40</td>
<td>-1.225</td>
<td>-1.281</td>
<td>0.431</td>
<td>0.540</td>
</tr>
<tr>
<td>0.75</td>
<td>-0.85</td>
<td>-0.475</td>
<td>-0.119</td>
<td>0.240</td>
<td>0.199</td>
</tr>
<tr>
<td>1.00</td>
<td>-0.50</td>
<td>-0.370</td>
<td>-0.098</td>
<td>0.150</td>
<td>0.083</td>
</tr>
<tr>
<td>1.25</td>
<td>-0.28</td>
<td>-0.025</td>
<td>-0.051</td>
<td>0.058</td>
<td>0.015</td>
</tr>
<tr>
<td>1.50</td>
<td>-0.13</td>
<td>0.024</td>
<td>0.060</td>
<td>0.010</td>
<td>0.005</td>
</tr>
<tr>
<td>1.75</td>
<td>-0.06</td>
<td>-0.054</td>
<td>0.006</td>
<td>0.004</td>
<td>0.001</td>
</tr>
<tr>
<td>2.00</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.064</td>
<td>0.089</td>
</tr>
</tbody>
</table>

(a) We had found \( v_0 = 1.914 \text{ m/s} \n
(b) At \( t = 2 \text{ s} \)

\( x = 0.840 \text{ m} \)
PROBLEM 11.83

A training airplane has a velocity of 38 m/s when it lands on an aircraft carrier. As the arresting gear of the carrier brings the airplane to rest, the velocity and the acceleration of the airplane are recorded; the results are shown (solid curve) in the figure. Determine by approximate means (a) the time required for the airplane to come to rest, (b) the distance traveled in that time.

SOLUTION

Given: $a$–$v$ curve:

$$v_0 = 38 \text{ m/s}$$

The given curve is approximated by a series of uniformly accelerated motions (the horizontal dashed lines on the figure).

For uniformly accelerated motion

$$v_2 = v_1 + a(t_2 - t_1)$$

or

$$\Delta x = \frac{v_2^2 - v_1^2}{2a}$$

$$\Delta t = \frac{v_2 - v_1}{a}$$

For the five regions shown above, we have

<table>
<thead>
<tr>
<th>Region</th>
<th>$v_1$, m/s</th>
<th>$v_2$, m/s</th>
<th>$a$, m/s$^2$</th>
<th>$\Delta x$, m</th>
<th>$\Delta t$, s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>38</td>
<td>36</td>
<td>-4</td>
<td>18.5</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>36</td>
<td>30</td>
<td>-10</td>
<td>19.8</td>
<td>0.6</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>24</td>
<td>-13.5</td>
<td>12</td>
<td>0.4444</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td>12</td>
<td>-16</td>
<td>13.5</td>
<td>0.75</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>0</td>
<td>-17</td>
<td>4.235</td>
<td>0.7059</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td></td>
<td></td>
<td></td>
<td>68.035</td>
<td>3.0003</td>
</tr>
</tbody>
</table>

(a) From the table, when $v = 0$

$$\Delta t = 3.00 \text{ s}$$

(b) From the table and assuming $x_0 = 0$, when $v = 0$

$$x = 68.0 \text{ m}$$
PROBLEM 11.84

Shown in the figure is a portion of the experimentally determined v–x curve for a shuttle cart. Determine by approximate means the acceleration of the cart (a) when \( x = 250 \text{ mm} \), (b) when \( v = 2000 \text{ mm/s} \).

\[ v(x) \]

\[ x(\text{mm}) \]

\[ 0 \quad 250 \quad 500 \quad 750 \quad 1000 \quad 1250 \]

\[ 0 \quad 500 \quad 1000 \quad 1500 \quad 2000 \]

SOLUTION

Given: \( v–x \) curve

First note that the slope of the above curve is \( \frac{dv}{dx} \). Now

\[ a = \frac{dv}{dx} \]

(a) When \( x = 250 \text{ mm}, \ v = 1375 \text{ mm/s} \)

Then

\[ a = (1375 \text{ mm/s}) \left( \frac{1000 \text{ m/s}}{337.5 \text{ mm}} \right) \]

or

\[ a = 4070 \text{ mm/s}^2 \]

(b) When \( v = 2000 \text{ mm/s} \), we have

\[ a = 2000 \text{ mm/s} \left( \frac{1000 \text{ m/s}}{700 \text{ mm}} \right) \]

or

\[ a = 2860 \text{ mm/s}^2 \]

Note: To use the method of measuring the subnormal outlined at the end of Section 11.8, it is necessary that the same scale be used for the x and v axes (e.g., 1 cm = 50 cm, 1 cm = 50 cm/s). In the above solution, \( \Delta v \) and \( \Delta x \) were measured directly, so different scales could be used.
PROBLEM 11.85

Using the method of Section 11.8, derive the formula \( x = x_0 + v_0 t + \frac{1}{2} at^2 \) for the position coordinate of a particle in uniformly accelerated rectilinear motion.

SOLUTION

The \( a-t \) curve for uniformly accelerated motion is as shown.

Using Eq. (11.13), we have

\[
x = x_0 + v_0 t + (\text{area under } a-t \text{ curve}) (t-\tau) \\
= x_0 + v_0 t + (t \times a) \left( t - \frac{1}{2} t \right) \\
= x_0 + v_0 t + \frac{1}{2} at^2
\]

Q.E.D.\(\blacksquare\)
PROBLEM 11.86
Using the method of Section 11.8, determine the position of the particle of Problem 11.61 when $t = 4$ s.

PROBLEM 11.61 A particle moves in a straight line with the acceleration shown in the figure. Knowing that it starts from the origin with $v_0 = -6$ m/s, (a) plot the $v$–$t$ and $x$–$t$ curves for $0 < t < 20$ s, (b) determine its velocity, its position, and the total distance traveled after 12 s.

SOLUTION

When $t = 14$ s:

$x = x_0 + v_0 t + \sum A(t_i - \bar{t})$

$= 0 - (6 \text{ m/s})(14 \text{ s}) + [(1 \text{ m/s}^2)(4 \text{ s})](12 \text{ s}) + [(2 \text{ m/s}^2)(6 \text{ s})](7 \text{ s}) + [(7 \text{ m/s}^2)(2 \text{ s})]$

$x_{14} = -84 \text{ m} + 48 \text{ m} + 84 \text{ m} - 16 \text{ m} = 32 \text{ m}$

$x_{14} = +32 \text{ m}$
**PROBLEM 11.87**

While testing a new lifeboat, an accelerometer attached to the boat provides the record shown. If the boat has a velocity of 2.5 m/s at \( t = 0 \) and is at rest at time \( t_1 \), determine, using the method of Section 11.8, (a) the time \( t_1 \), (b) the distance through which the boat moves before coming to rest.

**SOLUTION**

The area under the curve is divided into three regions as shown.

(a) First note
\[
\frac{t_a}{20} = \frac{0.75}{25}
\]
or
\[t_a = 0.60 \text{ s}\]
Now
\[
v = v_0 + \int_0^t a \, dt
\]
where the integral is equal to the area under the \( a-t \) curve. Then, with \( v_0 = 2.5 \text{ m/s}, v_t = 0 \)
We have
\[
0 = 2.5 \text{ m/s} + \left[ \frac{1}{2} (0.6 \text{ s})(20 \text{ m/s}^2) - \frac{1}{2} (0.15 \text{ s})(5 \text{ m/s}^2) - (t_1 - 0.75) \text{ s} \times (5 \text{ m/s}^2) \right]
\]
or
\[t_1 = 2.375 \text{ s} \quad t_1 = 2.38 \text{ s} \]

(b) From the discussion following Eq. (11.13) and assuming \( x = 0 \), we have
\[
x = 0 + v_0 t_1 + \Sigma A(t_1 - \bar{t})
\]
where \( A \) is the area of a region and \( \bar{t} \) is the distance to its centroid. Then for \( t_1 = 2.375 \text{ s} \)
\[
x = (2.5 \text{ m/s})(2.375 \text{ s}) + \left[ \frac{1}{2} (0.6 \text{ s})(20 \text{ m/s}^2) \right] (2.375 - 0.2) \text{ s}
- \left[ \frac{1}{2} (0.15 \text{ s})(5 \text{ m/s}^2) \right] (2.375 - 0.70) \text{ s}
- [(1.625 \text{ s})(5 \text{ m/s}^2)] \left[ 2.375 - \left( 0.75 + \frac{1}{2} \times 1.625 \right) \right] \text{ s}
\]
\[= [5.9375 + (13.05 - 0.6281 - 6.6016)] \text{ m}
\]
or
\[x = 11.76 \text{ m} \]

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PROBLEM 11.88

For the particle of Problem 11.63, draw the a–t curve and determine, using the method of Section 11.8, (a) the position of the particle when t = 52 s, (b) the maximum value of its position coordinate.

PROBLEM 11.63 A particle moves in a straight line with the velocity shown in the figure. Knowing that x = −180 m at t = 0, (a) construct the a–t and x–t curves for 0 < t < 50 s, and determine (b) the total distance traveled by the particle when t = 50 s, (c) the two times at which x = 0.

SOLUTION

We have

\[ a = \frac{dv}{dt} \]

where \( \frac{dv}{dt} \) is the slope of the v–t curve. Then

\[
\begin{align*}
t = 0 \text{ to } t = 10 \text{ s:} \quad & v = \text{constant} \Rightarrow a = 0 \\
t = 10 \text{ s} \text{ to } t = 26 \text{ s:} \quad & a = \frac{-10 - 20}{26 - 10} = \frac{-15}{8} \text{ m/s}^2 \\
t = 26 \text{ s} \text{ to } t = 41 \text{ s:} \quad & v = \text{constant} \Rightarrow a = 0 \\
t = 41 \text{ s} \text{ to } t = 46 \text{ s:} \quad & a = \frac{-4 - (-10)}{46 - 41} = -1.2 \text{ m/s}^2 \\
t > 46 \text{ s:} \quad & v = \text{constant} \Rightarrow a = 0
\end{align*}
\]

The a–t curve is then drawn as shown.

(a) From the discussion following Eq. (11.13), we have

\[ x = x_0 + v_0 t + \frac{1}{2}A(t_1 - \bar{t}) \]

where A is the area of a region and \( \bar{t} \) is the distance to its centroid. Then, for \( t_1 = 52 \text{ s} \)

\[
\begin{align*}
x &= -180 \text{ m} + (20 \text{ m/s})(52 \text{ s}) + \left\{\left(-\frac{15}{8} \text{ m/s}^2\right)(52 - 18) \text{ s}\right. \\
&\left.\quad + [5 \text{ s}(1.2 \text{ m/s}^2)](52 - 43.5 \text{ s})\right\} \\
&= -180 \text{ m} + 1040 \text{ m} + (-1020 \text{ m} + 51 \text{ m})
\end{align*}
\]

or

\[ x = -109 \text{ m} \]

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PROBLEM 11.88 (Continued)

(b) Noting that \( x_{\text{max}} \) occurs when \( v = 0 \left( \frac{dv}{dt} = 0 \right) \), it is seen from the \( v-t \) curve that \( x_{\text{max}} \) occurs for \( 10 \text{s} < t < 26 \text{s} \). Although similar triangles could be used to determine the time at which \( x = x_{\text{max}} \) (see the solution to Problem 11.63), the following method will be used.

For \( 10 \text{s} < t < 26 \text{s} \), we have

\[
x = -180 + 20t - \left[ (t - 10) \left( \frac{15}{8} \right) \right] \left[ \frac{1}{2} (t - 10) \right] \text{m}
\]

\[
= -180 + 20t - \frac{15}{16} (t - 10)^2
\]

When \( x = x_{\text{max}} \):

\[
\frac{dx}{dt} = 20 - \frac{30}{16} (t - 10) = 0
\]

or

\[
(t)_{x_{\text{max}}} = \frac{62}{3} \text{s}
\]

Then

\[
x_{\text{max}} = -180 + 20 \left( \frac{62}{3} \right) - \frac{15}{16} \left( \frac{62}{3} - 10 \right)^2
\]

or

\[
x_{\text{max}} = 126.7 \text{ m}
\]
**PROBLEM 11.89**

The motion of a particle is defined by the equations \( x = 4t^3 - 5t^2 + 5t \) and \( y = 5t^2 - 15t \), where \( x \) and \( y \) are expressed in millimeters and \( t \) is expressed in seconds. Determine the velocity and the acceleration when (a) \( t = 1 \) s, (b) \( t = 2 \) s.

**SOLUTION**

\[ x = 4t^3 - 5t^2 + 5t \quad y = 5t^2 - 15t \]

\[ v_x = \frac{dx}{dt} = 12t^2 - 10t + 5 \quad v_y = \frac{dy}{dt} = 10t - 15 \]

\[ a_x = \frac{dv_x}{dt} = 24t - 10 \quad a_y = \frac{dv_y}{dt} = 10 \]

(a) When \( t = 15 \), \( v_x = 7 \) mm/s, \( v_y = -5 \) mm/s

\[ v = \sqrt{v_x^2 + v_y^2} = 8.60 \quad \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right) = 35.5^\circ \]

\[ v = 8.60 \text{ mm/s} \rightarrow 35.5^\circ \]

\[ a_x = 14 \text{ mm/s}^2 \quad a_y = 10 \text{ mm/s}^2 \]

\[ a = \sqrt{a_x^2 + a_y^2} = 17.20 \text{ mm/s}^2 \]

\[ \phi = \tan^{-1}\left(\frac{10}{14}\right) = 35.5^\circ \]

\[ a = 17.20 \text{ mm/s}^2 \rightarrow 35.5^\circ \]

(b) When \( t = 25 \), \( v_x = 33 \) mm/s, \( v_y = 5 \) mm/s

\[ v = \sqrt{v_x^2 + v_y^2} = 33.4 \text{ mm/s} \rightarrow 8.62^\circ \]

\[ a_x = 38 \text{ mm/s}^2 \]

\[ a_y = 10 \text{ mm/s}^2 \]

\[ a = \sqrt{a_x^2 + a_y^2} = 39.3 \text{ mm/s}^2 \rightarrow 14.74^\circ \]

\[ a = 39.3 \text{ mm/s}^2 \rightarrow 14.74^\circ \]
PROBLEM 11.90

The motion of a particle is defined by the equations $x = 2 \cos \pi t$ and $y = 1 - 4 \cos 2\pi t$, where $x$ and $y$ are expressed in meters and $t$ is expressed in seconds. Show that the path of the particle is part of the parabola shown, and determine the velocity and the acceleration when (a) $t = 0$, (b) $t = 1.5$ s.

\[ y = 5 - 2x^2 \]

\[ x = 2 \cos \pi t \quad y = 1 - 4 \cos 2\pi t \]

\[ y = 1 - 4(2 \cos^2 \pi t - 1) \]

\[ = 5 - 8 \left( \frac{x^2}{2} \right) \]

\[ y = 5 - 2x^2 \quad \text{Q.E.D.} \]

\[ v_x = \frac{dx}{dt} = -2\pi \sin \pi t \quad v_y = \frac{dy}{dt} = 8\pi \sin 2\pi t \]

\[ a_x = \frac{dv_x}{dt} = -2\pi^2 \cos \pi t \quad a_y = \frac{dv_y}{dt} = 16\pi^2 \cos 2\pi t \]

(a) At $t = 0$: $v_x = 0$ \quad $v_y = 0$ \quad $v = 0$ \quad $a_x = -2\pi^2$ m/s$^2$ \quad $a_y = 16\pi^2$ m/s$^2$

or

(b) At $t = 1.53$: $v_x = -2\pi \sin(1.5\pi) = 2\pi$ m/s \quad $v_y = 8\pi \sin(2\pi \times 1.5) = 0$

or

\[ v = 6.28 \text{ m/s} \quad a = 159.1 \text{ m/s}^2 \quad \theta = 82.9^\circ \]

or

\[ a_x = -2\pi^2 \cos(1.5\pi) = 0 \quad a_y = 16\pi^2 \cos(2\pi \times 1.5) = -16\pi^2 \]

or

\[ a = 157.9 \text{ m/s}^2 \quad \theta = 82.9^\circ \]
PROBLEM 11.91

The motion of a particle is defined by the equations \( x = t^2 - 8t + 7 \) and \( y = 0.5t^2 + 2t - 4 \), where \( x \) and \( y \) are expressed in meters and \( t \) in seconds. Determine (a) the magnitude of the smallest velocity reached by the particle, (b) the corresponding time, position, and direction of the velocity.

SOLUTION

\[
\begin{align*}
x &= t^2 - 8t + 7 & y &= 0.5t^2 + 2t - 4 \\
v_x &= \frac{dx}{dt} = 2t - 8 & v_y &= t + 2 \\
a_x &= \frac{dv_x}{dt} = 2 & a_y &= \frac{dv_y}{dt} = 1 \\
v^2 &= v_x^2 + v_y^2 = (2t - 8)^2 + (t + 2)^2
\end{align*}
\]

When \( v \) is minimum, \( v^2 \) is also minimum; thus, we write

\[
\frac{d(v^2)}{dt} = 2(2t - 8)(2) + 2(t + 2) = 2(4t - 16 + t + 2) = 0 \\
= 2(5t - 14) = 0 \\
t = 2.80 \text{ s}
\]

When \( t = 2.8 \text{ s} \):

\[
\begin{align*}
x &= (2.8)^2 - 8(2.8) + 7 & x &= -7.56 \text{ m} \\
y &= 0.5(2.8)^2 + 2(2.8) - 4 & y &= 5.52 \text{ m} \\
v_x &= 2(2.8) - 8 & v_x &= -2.40 \text{ m/s} \\
v_y &= 2.8 + 2 & v_y &= +4.80 \text{ m/s} \\
v &= 5.37 \text{ m/s} \quad \rightarrow \quad 63.4^\circ
\end{align*}
\]
PROBLEM 11.92

The motion of a particle is defined by the equations \( x = 100t - 50 \sin t \) and \( y = 100 - 50 \cos t \), where \( x \) and \( y \) are expressed in mm and \( t \) is expressed in seconds. Sketch the path of the particle, and determine (a) the magnitudes of the smallest and largest velocities reached by the particle, (b) the corresponding times, positions, and directions of the velocities.

SOLUTION

We have \( x = 4t - 2 \sin t \quad y = 4 - 2 \cos t \)

<table>
<thead>
<tr>
<th>( t, ) s</th>
<th>( x, ) mm</th>
<th>( y, ) mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>( \pi/2 )</td>
<td>107.08</td>
<td>100</td>
</tr>
<tr>
<td>( \pi )</td>
<td>314.25</td>
<td>150</td>
</tr>
<tr>
<td>( 3\pi/2 )</td>
<td>521.24</td>
<td>100</td>
</tr>
<tr>
<td>( 2\pi )</td>
<td>628.32</td>
<td>50</td>
</tr>
</tbody>
</table>

(a) We have \( x = 100t - 50 \sin t = 25(4t - 2\sin t) \quad y = 100 - 50 \cos t = 25(4 - 2 \cos t) \)

Then \( v_x = \frac{dx}{dt} = 25(4 - 2 \cos t) \quad v_y = \frac{dy}{dt} = 50 \sin t \)

Now \( v^2 = v_x^2 + v_y^2 = 625(4 - 2 \cos t)^2 + 2500 \sin^2 t \)

= \( 12500 - 10000t \)

By observation,

for \( v_{\text{min}}, \cos t = 1 \), so that

\( v_{\text{min}}^2 = 2500 \quad \text{or} \quad v_{\text{min}} = 50 \text{ mm/s} \)

for \( v_{\text{max}}, \cos t = -1 \), so that

\( v_{\text{max}}^2 = 22500 \quad \text{or} \quad v_{\text{max}} = 150 \text{ mm/s} \)

(b) When \( v = v_{\text{min}} \):

\( \cos t = 1 \quad \text{or} \quad t = 2n\pi \text{ s} \)

Then \( x = 25(4(2n\pi) - 2 \sin (2n\pi)) \quad \text{or} \quad x = 200 n\pi \text{ mm} \)

\( y = 25(4 - 2(1)) = 50 \quad \text{or} \quad y = 50.0 \text{ mm} \)
PROBLEM 11.92 (Continued)

Also,

\[ v_x = 25 \{4 - 2(1)\} = 50 \text{ mm/s} \]
\[ v_y = 50 \sin (2n\pi) = 0 \]

\[ \theta_{v_{\text{min}}} = 0 \rightarrow \text{ } \blacktriangle \]

When \( v = v_{\text{max}} \):

\[ \cos t = -1 \]
\[ \text{or } t = (2n+1)\pi \text{ s } \blacktriangle \]
\[ \text{where } n = 0, 1, 2, \ldots \]

Then

\[ x = 25\{4(2n+1)\pi - 2\sin (2n+1)\pi\} \]
\[ y = 25\{4 - 2(-1)\} \]

\[ \text{or } x = 100(2n+1)\pi \text{ mm } \blacktriangle \]
\[ \text{or } y = 150 \text{ mm } \blacktriangle \]

Also,

\[ v_x = 25(4 - 2(-1)) = 150 \text{ mm/s} \]
\[ v_y = 50 \sin (2n+1)\pi = 0 \]

\[ \theta_{v_{\text{max}}} = 0 \rightarrow \text{ } \blacktriangle \]
**PROBLEM 11.93**

The motion of a particle is defined by the position vector \( \mathbf{r} = A(\cos t + t \sin t) \mathbf{i} + A(\sin t - t \cos t) \mathbf{j} \), where \( t \) is expressed in seconds. Determine the values of \( t \) for which the position vector and the acceleration are (a) perpendicular, (b) parallel.

**SOLUTION**

We have

\[
\mathbf{r} = A(\cos t + t \sin t) \mathbf{i} + A(\sin t - t \cos t) \mathbf{j}
\]

Then

\[
\mathbf{v} = \frac{d\mathbf{r}}{dt} = -A \sin t + A \cos t \mathbf{i} + (A \cos t - A \sin t + 2t \cos t) \mathbf{j}
\]

and

\[
\mathbf{a} = \frac{d\mathbf{v}}{dt} = -A \cos t - A \sin t \mathbf{i} + (A \sin t - A \cos t + 2 \cos t + 2t \sin t) \mathbf{j}
\]

(a) When \( \mathbf{r} \) and \( \mathbf{a} \) are perpendicular, \( \mathbf{r} \cdot \mathbf{a} = 0 \)

\[
A((\cos t + t \sin t) \mathbf{i} + (\sin t - t \cos t) \mathbf{j}) \cdot A((\cos t - t \sin t) \mathbf{i} + (\sin t + t \cos t) \mathbf{j}) = 0
\]

or

\[
(\cos t + t \sin t)(\cos t - t \sin t) + (\sin t - t \cos t)(\sin t + t \cos t) = 0
\]

or

\[
(\cos^2 t - t^2 \sin^2 t) + (\sin^2 t - t^2 \cos^2 t) = 0
\]

or

\[
1 - t^2 = 0 \quad \text{or} \quad t = 1 \text{s} \downarrow
\]

(b) When \( \mathbf{r} \) and \( \mathbf{a} \) are parallel, \( \mathbf{r} \times \mathbf{a} = 0 \)

\[
A((\cos t + t \sin t) \mathbf{i} + (\sin t - t \cos t) \mathbf{j}) \times A((\cos t - t \sin t) \mathbf{i} + (\sin t + t \cos t) \mathbf{j}) = 0
\]

or

\[
[(\cos t + t \sin t)(\sin t + t \cos t) - (\sin t - t \cos t)(\cos t - t \sin t)] \mathbf{k} = 0
\]

Expanding

\[
(\sin t \cos t + t^2 \sin t \cos t) - (\sin t \cos t - t^2 \sin t \cos t) = 0
\]

or

\[
2t^2 = 0 \quad \text{or} \quad t = 0 \downarrow
\]
PROBLEM 11.94

The damped motion of a vibrating particle is defined by the position vector \( \mathbf{r} = x_1(1 - 1/(t + 1)) \mathbf{i} + y_1 e^{-\pi t/2} \cos 2\pi t \mathbf{j} \), where \( t \) is expressed in seconds. For \( x_1 = 30 \text{ mm} \) and \( y_1 = 20 \text{ mm} \), determine the position, the velocity, and the acceleration of the particle when (a) \( t = 0 \), (b) \( t = 1.5 \text{ s} \).

SOLUTION

We have

\[
\mathbf{r} = 30 \left( 1 - \frac{1}{t + 1} \right) \mathbf{i} + 20(e^{-\pi t/2} \cos 2\pi t) \mathbf{j}
\]

Then

\[
\mathbf{v} = \frac{d\mathbf{r}}{dt} = 30 \cdot \frac{1}{(t + 1)^2} \mathbf{i} + 20 \left( -\frac{\pi}{2} e^{-\pi t/2} \cos 2\pi t - 2\pi e^{-\pi t/2} \sin 2\pi t \right) \mathbf{j}
\]

and

\[
\mathbf{a} = \frac{d\mathbf{v}}{dt} = -30 \cdot \frac{2}{(t + 1)^3} \mathbf{i} - 20\pi \left[ \frac{\pi}{2} e^{-\pi t/2} \left( \frac{1}{2} \cos 2\pi t + 2 \sin 2\pi t \right) + e^{-\pi t/2} (-\pi \sin 2\pi t + 4 \cos 2\pi t) \right] \mathbf{j}
\]

(a) At \( t = 0 \):

\[
\mathbf{r} = 30 \left( 1 - \frac{1}{1} \right) \mathbf{i} + 20(1) \mathbf{j} \quad \text{or} \quad \mathbf{r} = 20 \text{ mm} \uparrow \\
\mathbf{v} = 30 \mathbf{i} - 20\pi \left( \frac{1}{1} \left( \frac{1}{2} + 0 \right) \right) \mathbf{j} \quad \text{or} \quad \mathbf{v} = 43.4 \text{ mm/s} \quad 46.3^\circ \\
\mathbf{a} = -\frac{60}{1} \mathbf{i} + 10\pi^2 (1)(0 - 7.5) \mathbf{j} \quad \text{or} \quad \mathbf{a} = 743 \text{ mm/s}^2 \quad 85.4^\circ.
PROBLEM 11.94 (Continued)

(b) At \( t = 1.5 \) s:

\[
\mathbf{r} = 30 \left( 1 - \frac{1}{2.5} \right) \mathbf{i} + 20e^{-0.75\pi (\cos 3\pi)} \mathbf{j} = (18 \text{ mm})\mathbf{i} + (-1.8956 \text{ mm})\mathbf{j}
\]

or

\[
\mathbf{r} = 18.10 \text{ mm} \quad \theta = 6.01^\circ\quad \downarrow
\]

\[
\mathbf{v} = \frac{30}{(2.5)^2} \mathbf{i} - 20\pi e^{-0.75\pi} \left( \frac{1}{2} \cos 3\pi + 0 \right) \mathbf{j} = (4.80 \text{ mm/s})\mathbf{i} + (2.9778 \text{ mm/s})\mathbf{j}
\]

or

\[
\mathbf{v} = 5.65 \text{ mm/s} \quad \theta = 31.8^\circ\quad \downarrow
\]

\[
\mathbf{a} = -\frac{60}{(2.5)^3} \mathbf{i} + 10\pi^2 e^{-0.75\pi} (0 - 7.5 \cos 3\pi) \mathbf{j} = (-3.84 \text{ mm/s}^2)\mathbf{i} + (70.1582 \text{ mm/s}^2)\mathbf{j}
\]

or

\[
\mathbf{a} = 70.3 \text{ mm/s}^2 \quad \theta = 86.9^\circ\quad \downarrow
\]
PROBLEM 11.95

The three-dimensional motion of a particle is defined by the position vector \( \mathbf{r} = (Rt \cos \omega_n t) \mathbf{i} + ct \mathbf{j} + (Rt \sin \omega_n t) \mathbf{k} \). Determine the magnitudes of the velocity and acceleration of the particle. (The space curve described by the particle is a conic helix.)

SOLUTION

We have

\[ \mathbf{r} = (Rt \cos \omega_n t) \mathbf{i} + ct \mathbf{j} + (Rt \sin \omega_n t) \mathbf{k} \]

Then

\[ \mathbf{v} = \frac{d\mathbf{r}}{dt} = R(\cos \omega_n t - \omega_n t \sin \omega_n t) \mathbf{i} + c \mathbf{j} + R(\sin \omega_n t + \omega_n t \cos \omega_n t) \mathbf{k} \]

and

\[ \mathbf{a} = \frac{d\mathbf{v}}{dt} = R(-\omega_n \sin \omega_n t - \omega_n \sin \omega_n t - \omega_n^2 t \cos \omega_n t) \mathbf{i} + \omega_n \cos \omega_n t + \omega_n \cos \omega_n t - \omega_n^2 t \sin \omega_n t) \mathbf{k} \]

Now

\[ v^2 = v_x^2 + v_y^2 + v_z^2 \]

\[ = [R(\cos \omega_n t - \omega_n t \sin \omega_n t)]^2 + (c)^2 + [R(\sin \omega_n t + \omega_n t \cos \omega_n t)]^2 \]

\[ = R^2 \left[ \left( \cos^2 \omega_n t - 2\omega_n t \sin \omega_n t \cos \omega_n t + \omega_n^2 t^2 \sin^2 \omega_n t \right) \right] \]

\[ + \left( \sin^2 \omega_n t + 2\omega_n t \sin \omega_n t \cos \omega_n t + \omega_n^2 t^2 \cos^2 \omega_n t \right) \]

\[ = R^2 \left( 1 + \omega_n^2 t^2 \right) + c^2 \]

or

\[ v = \sqrt{R^2 (1 + \omega_n^2 t^2) + c^2} \]

Also,

\[ a^2 = a_x^2 + a_y^2 + a_z^2 \]

\[ = \left[ R(-2\omega_n \sin \omega_n t - \omega_n^2 t \cos \omega_n t) \right]^2 + (0) \]

\[ + \left[ R(2\omega_n \cos \omega_n t - \omega_n^2 t \sin \omega_n t) \right]^2 \]

\[ = R^2 \left[ \left( 4\omega_n^2 \sin^2 \omega_n t + 4\omega_n^3 t \sin \omega_n t \cos \omega_n t + \omega_n^4 t^2 \cos^2 \omega_n t \right) \right. \]

\[ + \left. \left( 4\omega_n^2 \cos^2 \omega_n t - 4\omega_n^3 t \sin \omega_n t \cos \omega_n t + \omega_n^4 t^2 \sin^2 \omega_n t \right) \right] \]

\[ = R^2 \left( 4\omega_n^2 + \omega_n^4 t^2 \right) \]

or

\[ a = R\omega_n \sqrt{4 + \omega_n^2 t^2} \]
PROBLEM 11.96*

The three-dimensional motion of a particle is defined by the position vector \( \mathbf{r} = (At \cos t)\mathbf{i} + (At^2 + 1)\mathbf{j} + (Bt \sin t)\mathbf{k} \), where \( r \) and \( t \) are expressed in feet and seconds, respectively. Show that the curve described by the particle lies on the hyperboloid \( \left(\frac{y}{A}\right)^2 - \left(\frac{x}{A}\right)^2 - \left(\frac{z}{B}\right)^2 = 1 \). For \( A = 3 \) and \( B = 1 \), determine (a) the magnitudes of the velocity and acceleration when \( t = 0 \), (b) the smallest nonzero value of \( t \) for which the position vector and the velocity are perpendicular to each other.

SOLUTION

We have \( \mathbf{r} = (At \cos t)\mathbf{i} + (At^2 + 1)\mathbf{j} + (Bt \sin t)\mathbf{k} \)

or

\( x = At \cos t \quad y = At^2 + 1 \quad z = Bt \sin t \)

Then

\[ \cos^2 t + \sin^2 t = 1 \Rightarrow \left(\frac{x}{At}\right)^2 + \left(\frac{z}{Bt}\right)^2 = 1 \]

Now

\[ \cos^2 t + \sin^2 t = 1 \Rightarrow \left(\frac{x}{At}\right)^2 + \left(\frac{z}{Bt}\right)^2 = 1 \]

or

\[ t^2 = \left(\frac{x}{A}\right)^2 + \left(\frac{z}{B}\right)^2 \]

Then

\[ \left(\frac{y}{A}\right)^2 - \left(\frac{x}{A}\right)^2 - \left(\frac{z}{B}\right)^2 = 1 \] Q.E.D.

(a) With \( A = 3 \) and \( B = 1 \), we have

\[ \mathbf{v} = \frac{d\mathbf{r}}{dt} = 3(\cos t - t \sin t)\mathbf{i} + \frac{3t}{\sqrt{t^2 + 1}}\mathbf{j} + (\sin t + t \cos t)\mathbf{k} \]

and

\[ \mathbf{a} = \frac{d\mathbf{v}}{dt} = 3(-\sin t - t \cos t)\mathbf{i} + \frac{3\sqrt{t^2 + 1} - t \left(\frac{1}{\sqrt{t^2 + 1}}\right)}{(t^2 + 1)^{3/2}}\mathbf{j} + (\cos t + t \sin t)\mathbf{k} \]

\[ = -3(2 \sin t + t \cos t)\mathbf{i} + \frac{3}{(t^2 + 1)^{3/2}}\mathbf{j} + (2 \cos t - \sin t)\mathbf{k} \]
PROBLEM 11.96* (Continued)

At $t = 0$:

$v = 3(1-0)i + (0)j + (0)k$

$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

or $v = 3 \text{ m/s}$

and $a = -3(0)i + 3(1)j + (2-0)k$

Then $a^2 = (0)^2 + (3)^2 + (2)^2 = 13$

or $a = 3.61 \text{ m/s}^2$

(b) If $\mathbf{r}$ and $\mathbf{v}$ are perpendicular, $\mathbf{r} \cdot \mathbf{v} = 0$

$[(3 \cos t)i + (3\sqrt{t^2+1})j + t \sin t)k] \cdot [(3\cos t - t \sin t)i + \left(3 - \frac{t}{\sqrt{t^2+1}}\right)j + (\sin t + t \cos t)k] = 0$

or $[(3 \cos t)[3(\cos t - t \sin t)] + (3\sqrt{t^2+1})\left(3 - \frac{t}{\sqrt{t^2+1}}\right) + (t \sin t)(\sin t + t \cos t) = 0$

Expanding $(9t \cos^2 t - 9t^2 \sin t \cos t) + (9t) + (t \sin^2 t + t^2 \sin t \cos t) = 0$

or (with $t \neq 0$) $10 + 8 \cos^2 t - 8 \sin t \cos t = 0$

or $7 + 2 \cos 2t - 2 \sin 2t = 0$

Using “trial and error” or numerical methods, the smallest root is $t = 3.82 \text{ s}$

Note: The next root is $t = 4.38 \text{ s}$.
PROBLEM 11.97

An airplane used to drop water on brushfires is flying horizontally in a straight line at 315 km/h at an altitude of 80 m. Determine the distance d at which the pilot should release the water so that it will hit the fire at B.

SOLUTION

First note 

\[ v_0 = 315 \text{ km/h} = 87.5 \text{ m/s} \]

Vertical motion, (Uniformly accelerated motion)

\[ y = 0 + (0)t - \frac{1}{2}gt^2 \]

At B:

\[ -80 \text{ m} = -\frac{1}{2}(9.81 \text{ m/s}^2)t^2 \]

or

\[ t_B = 4.03855 \text{ s} \]

Horizontal motion, (Uniform)

\[ x = 0 + (v_x)_0t \]

At B:

\[ d = (87.5 \text{ m/s})(4.03855 \text{ s}) \]

or

\[ d = 353 \text{ m} \]
PROBLEM 11.98

Three children are throwing snowballs at each other. Child A throws a snowball with a horizontal velocity $v_0$. If the snowball just passes over the head of child B and hits child C, determine (a) the value of $v_0$, (b) the distance $d$.

SOLUTION

(a) \textbf{Vertical motion. (Uniformly accelerated motion)}

\[ y = 0 + (0)t - \frac{1}{2}gt^2 \]

At B:

\[ -1 \text{ m} = -\frac{1}{2}(9.81 \text{ m/s}^2)t^2 \quad \text{or} \quad t_B = 0.451524 \text{ s} \]

\textbf{Horizontal motion (Uniform)}

\[ x = 0 + (v_x\ )_0t \]

At B:

\[ 7 \text{ m} = v_0(0.451524 \text{ s}) \]

or

\[ v_0 = 15.5031 \text{ m/s} \]

(b) \textbf{Vertical motion: At C:}

\[ -3 \text{ m} = -\frac{1}{2}(9.81 \text{ m/s}^2)t^2 \]

or

\[ t_C = 0.782062 \text{ s} \]

\textbf{Horizontal motion.}

At C:

\[ (7 + d) \text{ m} = (15.5031 \text{ m/s})(0.782062 \text{ s}) \]

or

\[ d = 5.12 \text{ m} \]
PROBLEM 11.99

While delivering newspapers, a girl throws a newspaper with a horizontal velocity \( v_0 \). Determine the range of values of \( v_0 \) if the newspaper is to land between Points B and C.

SOLUTION

**Vertical motion.** (Uniformly accelerated motion)

\[
y = 0 + (0)t - \frac{1}{2}gt^2
\]

**Horizontal motion.** (Uniform)

\[
x = 0 + (v_0)t = v_0t
\]

At B:

\[
y: -1 \text{ m} = -\frac{1}{2}(9.81 \text{ m/s}^2)t^2
\]

or

\[
t_B = 0.45152 \text{ s}
\]

Then

\[
x: (2.1) \text{ m} = (v_0)_B(0.45152 \text{ s})
\]

or

\[
(v_0)_B = 4.651 \text{ m/s}
\]

At C:

\[
y: -0.6 \text{ m} = -\frac{1}{2}(9.81 \text{ m/s}^2)t^2
\]

or

\[
t_C = 0.34975 \text{ s}
\]

Then

\[
x: 3.7 \text{ m} = (v_0)_C(0.34975 \text{ s})
\]

or

\[
(v_0)_C = 10.579 \text{ m/s}
\]

\[
4.65 \text{ m/s} \leq v_0 \leq 10.58 \text{ m/s}
\]
**PROBLEM 11.100**

A baseball pitching machine “throws” baseballs with a horizontal velocity $v_0$. Knowing that height $h$ varies between 775 mm and 1050 mm, determine (a) the range of values of $v_0$, (b) the values of $\alpha$ corresponding to $h = 775$ mm and $h = 1050$ mm.

---

**SOLUTION**

(a) **Vertical motion.** (Uniformly accelerated motion)

$$y = 0 + (0)t - \frac{1}{2} gt^2$$

**Horizontal motion.** (Uniform)

$$x = 0 + (v_x)_0 t = v_0 t$$

When $h = 775$ mm, $y = -0.725$ m:

$$-0.725 = -\frac{1}{2} (9.81 \text{ m/s}^2) t^2$$

or

$$t_{31} = 0.38446 \text{ s}$$

Then

$$12 \text{ m} = (v_0)_{775} (0.38446 \text{ s})$$

or

$$(v_0)_{775} = 31.2126 \text{ m/s} = 112.365 \text{ km/h}$$

When $h = 1050$ mm, $y = -0.45$ m:

$$-0.45 = -\frac{1}{2} (9.81 \text{ m/s}^2) t^2$$

or

$$t_{42} = 0.30289 \text{ s}$$

Then

$$12 \text{ m} = (v_0)_{1050} (0.30289 \text{ s})$$

or

$$(v_0)_{1050} = 39.6183 \text{ m/s} = 142.626 \text{ km/h}$$

$$112.4 \text{ km/h} \leq v_0 \leq 142.6 \text{ km/h}$$

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(b) For the vertical motion

\[ v_y = (0) - gt \]

Now

\[ \tan \alpha = \left| \frac{(v_y)_B}{(v_y)_A} \right| = \frac{gt}{v_0} \]

When \( h = 775 \text{ mm} \):

\[ \tan \alpha = \frac{(9.81 \text{ m/s}^2)(0.38446 \text{ s})}{31.226 \text{ m/s}} = 0.12078 \]

or \( \alpha_{31} = 6.89^\circ \)

When \( h = 1050 \text{ mm} \):

\[ \tan \alpha = \frac{(9.81 \text{ m/s}^2)(0.30289 \text{ s})}{39.6183 \text{ m/s}} = 0.074999 \]

or \( \alpha_{42} = 4.29^\circ \)
PROBLEM 11.101
A volleyball player serves the ball with an initial velocity \( v_0 \) of magnitude 13.40 m/s at an angle of 20° with the horizontal. Determine (a) if the ball will clear the top of the net, (b) how far from the net the ball will land.

SOLUTION
First note

\[
\begin{align*}
(v_x)_0 &= (13.40 \text{ m/s}) \cos 20° = 12.5919 \text{ m/s} \\
(v_y)_0 &= (13.40 \text{ m/s}) \sin 20° = 4.5831 \text{ m/s}
\end{align*}
\]

(a) Horizontal motion. (Uniform)

\[
x = 0 + (v_x)_0 t
\]

At C

\[
9 \text{ m} = (12.5919 \text{ m/s}) t \quad \text{or} \quad t_C = 0.71475 \text{ s}
\]

Vertical motion. (Uniformly accelerated motion)

\[
y = y_0 + (v_y)_0 t - \frac{1}{2} gt^2
\]

At C:

\[
y_C = 2.1 \text{ m} + (4.5831 \text{ m/s})(0.71475 \text{ s}) - \frac{1}{2}(9.81 \text{ m/s}^2)(0.71475 \text{ s})^2 = 2.87 \text{ m}
\]

\[y_C > 2.43 \text{ m} \quad \text{(height of net)} \Rightarrow \text{ball clears net} \]

(b) At B, \( y = 0 \):

\[
0 = 2.1 \text{ m} + (4.5831 \text{ m/s}) t - \frac{1}{2}(9.81 \text{ m/s}^2) t^2
\]

Solving

\[
t_B = 1.271175 \text{ s} \quad \text{(the other root is negative)}
\]

Then

\[
d = (v_x)_0 t_B = (12.5919 \text{ m/s})(1.271175 \text{ s}) = 16.01 \text{ m}
\]

The ball lands

\[
b = (16.01 - 9.00) \text{ m} = 7.01 \text{ m} \text{ from the net} \]
PROBLEM 11.102

Milk is poured into a glass of height 140 mm and inside diameter 66 mm. If the initial velocity of the milk is 1.2 m/s at an angle of 40° with the horizontal, determine the range of values of the height h for which the milk will enter the glass.

SOLUTION

First note

\[(v_x)_0 = (1.2 \text{ m/s}) \cos 40° = 0.91925 \text{ m/s}\]
\[(v_y)_0 = -(1.2 \text{ m/s}) \sin 40° = -0.77135 \text{ m/s}\]

Horizontal motion. (Uniform)

\[x = 0 + (v_x)_0 t\]

Vertical motion. (Uniformly accelerated motion)

\[y = y_0 + (v_y)_0 t - \frac{1}{2} gt^2\]

Milk enters glass at B

\[x = 0.08 \text{ m} = (0.91925 \text{ m/s}) t \quad \text{or} \quad t_B = 0.087028 \text{ s}\]
\[y = 0.140 \text{ m} = h_B + (-0.77135 \text{ m/s})(0.087028 \text{ s}) - \frac{1}{2}(9.81 \text{ m/s}^2)(0.087028 \text{ s})^2\]

or

\[h_B = 0.244 \text{ m}\]

Milk enters glass at C

\[x = 0.146 \text{ m} = (0.91925 \text{ m/s}) t \quad \text{or} \quad t_C = 0.158825 \text{ s}\]
\[y = 0.140 \text{ m} = h_C + (-0.77135 \text{ m/s})(0.158825 \text{ s}) - \frac{1}{2}(9.81 \text{ m/s}^2)(0.158825 \text{ s})^2\]

or

\[h_C = 0.386 \text{ m}\]

\[0.244 \text{ m} \leq h \leq 0.386 \text{ m}\]
PROBLEM 11.103

A golfer hits a golf ball with an initial velocity of 50 m/s at an angle of 25° with the horizontal. Knowing that the fairway slopes downward at an average angle of 5°, determine the distance \( d \) between the golfer and Point B where the ball first lands.

SOLUTION

First note

\[
(v_x)_0 = (50 \text{ m/s}) \cos 25° \\
(v_y)_0 = (50 \text{ m/s}) \sin 25°
\]

and at B

\[
x_B = d \cos 5° \\
y_B = -d \sin 5°
\]

Now

**Horizontal motion.** (Uniform)

\[
x = 0 + (v_x)_0 t
\]

At B

\[
d \cos 5° = (50 \cos 25°) t \quad \text{or} \quad t_B = \frac{\cos 5°}{50 \cos 25°} d
\]

**Vertical motion.** (Uniformly accelerated motion)

\[
y = 0 + (v_y)_0 t - \frac{1}{2} g t^2 \quad (g = 9.81 \text{ m/s}^2)
\]

At B:

\[
-d \sin 5° = (50 \sin 25°) t_B - \frac{1}{2} g t_B^2
\]

Substituting for \( t_B \)

\[
-d \sin 5° = (50 \sin 25°) \left( \frac{\cos 5°}{50 \cos 25°} \right) d - \frac{1}{2} g \left( \frac{\cos 5°}{50 \cos 25°} \right)^2 d^2
\]

or

\[
d = \frac{2}{9.81 \cos 5°} (50 \cos 25°)^2 (\tan 5° + \tan 25°)
\]

or

\[d = 233 \text{ m} \quad \blacksquare\]
**PROBLEM 11.104**

Water flows from a drain spout with an initial velocity of 0.75 m/s at an angle of 15° with the horizontal. Determine the range of values of the distance \( d \) for which the water will enter the trough BC.

**SOLUTION**

First note

\[
(v_x)_0 = (0.75 \text{ m/s}) \cos 15° = 0.72444 \text{ m/s} \\
(v_y)_0 = -(0.75 \text{ m/s}) \sin 15° = -0.19411 \text{ m/s}
\]

**Vertical motion.** (Uniformly accelerated motion)

\[
y = 0 + (v_y)_0 t - \frac{1}{2} gt^2
\]

At the top of the trough, \( y = -3 + 0.36 = -2.64 \text{ m} \)

\[-2.64 \text{ m} = (-0.19411 \text{ m/s})t - \frac{1}{2}(9.81 \text{ m/s}^2)t^2 \]

or

\[t_{BC} = 0.71412 \text{ s} \quad \text{(the other root is negative)}
\]

**Horizontal motion.** (Uniform)

\[x = 0 + (v_x)_0 t\]

In time \( t_{BC} \)

\[x_{BC} = (0.72444 \text{ m/s})(0.71412 \text{ s}) = 0.5173 \text{ m}\]

Thus, the trough must be placed so that

\[x_B \leq 0.5173 \text{ m} \quad x_C \geq 0.5173 \text{ m}\]

Since the trough is 0.6 m wide, it then follows that

\[0 \leq d \leq 0.5173 \text{ m}\]
**PROBLEM 11.105**

Sand is discharged at A from a conveyor belt and falls onto the top of a stockpile at B. Knowing that the conveyor belt forms an angle $\alpha = 20^\circ$ with the horizontal, determine the speed $v_0$ of the belt.

**SOLUTION**

\[ (v_0)_x = v_0 \cos 20^\circ = 0.9397 v_0 \]
\[ (v_0)_y = v_0 \sin 20^\circ = 0.3420 v_0 \]

**Horizontal motion.**

\[ x = (v_0)_x t = (0.9397 v_0) t \]

At B:

\[ 10 \text{ m} = (0.9397 v_0) t \quad t = \frac{10.6417}{v_0} \tag{1} \]

**Vertical motion.**

\[ y = (v_0)_y t - \frac{1}{2} g t^2 \]

At B:

\[ -6 \text{ m} = v_0 (0.3420) t - \frac{1}{2} \cdot 9.81 t^2 \]

Using Eq. (1):

\[ -6 = (0.3420)(10.6417) - (4.905) \left( \frac{10.6417}{v_0} \right)^2 \]

\[ -9.6395 = -(4.905) \left( \frac{10.6417}{v_0} \right)^2 \]

\[ v_0^2 = \frac{(4.905)(10.6417)^2}{9.6395} = 57.6244 \text{ m}^2 / \text{s}^2 \]

\[ v_0 = 7.5911 \text{ m/s} \quad v_0 = 7.59 \text{ m/s} \]
PROBLEM 11.106

A basketball player shoots when she is 5 m from the backboard. Knowing that the ball has an initial velocity $v_0$ at an angle of $30^\circ$ with the horizontal, determine the value of $v_0$ when $d$ is equal to (a) 225 mm, (b) 425 mm.

SOLUTION

First note

$$(v_x)_0 = v_0 \cos 30^\circ$$
$$(v_y)_0 = v_0 \sin 30^\circ$$

Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

At $B$:

$$(5 - d) = (v_0 \cos 30^\circ) t$$  or  $$t_B = \frac{5 - d}{v_0 \cos 30^\circ}$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2} g t^2$$  (g = 9.81 m/s$^2$)

At $B$:

$$0.9 = (v_0 \sin 30^\circ) t_B - \frac{1}{2} g t_B^2$$

Substituting for $t_B$

$$0.9 = (v_0 \sin 30^\circ) \left( \frac{5 - d}{v_0 \cos 30^\circ} \right) - \frac{1}{2} g \left( \frac{5 - d}{v_0 \cos 30^\circ} \right)^2$$

or

$$v_0^2 = \frac{2g(5 - d)^2}{3} \left( \frac{1}{\sqrt{3}} (5 - d) - 0.9 \right)$$
PROBLEM 11.106 (Continued)

(a) \( d = 225 \text{ mm} = 0.225 \text{ m} \)
\[
v_0^2 = \frac{2(9.81)(5 - 0.225)^2}{3 \left[ \frac{1}{\sqrt{3}} (5 - 0.225) - 0.9 \right]}
\]
or
\[
v_0 = 8.96 \text{ m/s}
\]

(b) \( d = 425 \text{ mm} = 0.425 \text{ m} \)
\[
v_0^2 = \frac{2(9.81)(5 - 0.425)^2}{3 \left[ \frac{1}{\sqrt{3}} (5 - 0.425) - 0.9 \right]}
\]
or
\[
v_0 = 8.87 \text{ m/s}
\]
PROBLEM 11.107

A group of children are throwing balls through a 0.72-m-inner-diameter tire hanging from a tree. A child throws a ball with an initial velocity \( v_0 \) at an angle of 3\(^\circ\) with the horizontal. Determine the range of values of \( v_0 \) for which the ball will go through the tire.

SOLUTION

First note

\[
\begin{align*}
(v_x)_0 &= v_0 \cos 3^\circ \\
(v_y)_0 &= v_0 \sin 3^\circ
\end{align*}
\]

**Horizontal motion** (Uniform)

\[
x = 0 + (v_x)_0 t
\]

When \( x = 6 \) m:

\[
6 = (v_0 \cos 3^\circ) t \quad \text{or} \quad t_6 = \frac{6}{v_0 \cos 3^\circ}
\]

**Vertical motion** (Uniformly accelerated motion)

\[
y = 0 + (v_y)_0 t - \frac{1}{2} gt^2 \quad (g = 9.81 \text{ m/s}^2)
\]

When the ball reaches the tire, \( t = t_6 \)

\[
y_{B,C} = (v_0 \sin 3^\circ) \left( \frac{6}{v_0 \cos 3^\circ} \right) - \frac{1}{2} \left( \frac{6}{v_0 \cos 3^\circ} \right)^2
\]

or

\[
\frac{v_0^2}{v_0^2} = \frac{18(9.81)}{\cos^2 3^\circ(6 \tan 3^\circ - y_{B,C})}
\]

or

\[
\frac{v_0^2}{v_0^2} = \frac{177.065}{0.314447 - y_{B,C}}
\]

At \( B, \ y = -0.53 \) m:

\[
\frac{v_0^2}{v_0^2} = \frac{177.065}{0.314447 - (-0.53)}
\]

or

\[
(v_0)_B = 14.48 \text{ m/s}
\]
PROBLEM 11.107 (Continued)

At C, \(y = -1.25\) m:

\[
v_0^2 = \frac{177.06\text{s}}{0.314447 - (-1.25)}
\]

or

\[
(v_0)_C = 10.64 \text{ m/s}
\]

10.64 m/s \(\leq v_0 \leq 14.48\) m/s
**PROBLEM 11.108**

The nozzle at A discharges cooling water with an initial velocity $\mathbf{v}_0$ at an angle of $6^\circ$ with the horizontal onto a grinding wheel 350 mm in diameter. Determine the range of values of the initial velocity for which the water will land on the grinding wheel between Points B and C.

**SOLUTION**

First note

\[
\begin{align*}
(v_x)_0 &= v_0 \cos 6^\circ \\
(v_y)_0 &= -v_0 \sin 6^\circ
\end{align*}
\]

**Horizontal motion. (Uniform)**

\[x = x_0 + (v_x)_0 t\]

**Vertical motion. (Uniformly accelerated motion)**

\[y = y_0 + (v_y)_0 t - \frac{1}{2} gt^2 \quad (g = 9.81 \text{ m/s}^2)\]

At Point B:

\[x = (0.175 \text{ m}) \sin 10^\circ \]
\[y = (0.175 \text{ m}) \cos 10^\circ\]

\[x: \quad 0.175 \sin 10^\circ = -0.020 + (v_0 \cos 6^\circ)t\]

or

\[t_B = \frac{0.050388}{v_0 \cos 6^\circ}\]

\[y: \quad 0.175 \cos 10^\circ = 0.205 + (-v_0 \sin 6^\circ)t_B - \frac{1}{2} gt_B^2\]

Substituting for $t_B$

\[-0.032659 = (-v_0 \sin 6^\circ)\left(\frac{0.050388}{v_0 \cos 6^\circ}\right) - \frac{1}{2} (9.81)\left(\frac{0.050388}{v_0 \cos 6^\circ}\right)^2\]
PROBLEM 11.108 (Continued)

or \[ v_0^2 = \frac{1}{2} \frac{(9.81)(0.050388)^2}{\cos^2 6^\circ(0.032659 - 0.050388 \tan 6^\circ)} \]

or \( (v_0)_B = 0.678 \text{ m/s} \)

At Point C: \[
\begin{align*}
x &= (0.175 \text{ m}) \cos 30^\circ \\
y &= (0.175 \text{ m}) \sin 30^\circ \\
x &= 0.175 \cos 30^\circ = -0.020 + (v_0 \cos 6^\circ)t \\
\end{align*}
\]

or \( t_c = \frac{0.171554}{v_0 \cos 6^\circ} \)

\[
\begin{align*}
y &= 0.175 \sin 30^\circ = 0.205 + (-v_0 \sin 6^\circ)t_c - \frac{1}{2} gt_c^2 \\
\end{align*}
\]

Substituting for \( t_c \)

\[
-0.117500 = (-v_0 \sin 6^\circ) \left( \frac{0.171554}{v_0 \cos 6^\circ} \right) - \frac{1}{2} \left( \frac{(9.81)(0.171554)^2}{v_0 \cos 6^\circ} \right) - \frac{1}{2} \left( \frac{0.171554}{v_0 \cos 6^\circ} \right)^2
\]

or \[ v_0^2 = \frac{1}{2} \frac{(9.81)(0.171554)^2}{\cos^2 6^\circ(0.1175000 - 0.171554 \tan 6^\circ)} \]

or \( (v_0)_C = 1.211 \text{ m/s} \)

\[ 0.678 \text{ m/s} \leq v_0 \leq 1.211 \text{ m/s} \]
Problem 11.109

While holding one of its ends, a worker lobs a coil of rope over the lowest limb of a tree. If he throws the rope with an initial velocity $v_0$ at an angle of $65^\circ$ with the horizontal, determine the range of values of $v_0$ for which the rope will go over only the lowest limb.

Solution

First note

$$(v_x)_0 = v_0 \cos 65^\circ$$
$$(v_y)_0 = v_0 \sin 65^\circ$$

Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

At either B or C, $x = 5$ m

$$s = (v_x)_{0} \cos 65^\circ) t_{B,C}$$

or

$$t_{B,C} = \frac{5}{(v_0 \cos 65^\circ)}$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2} g t^2$$  \quad (g = 9.81 \text{ m/s}^2)

At the tree limbs, $t = t_{B,C}$

$$y_{B,C} = (v_y)_{0} \sin 65^\circ \left( \frac{5}{(v_0 \cos 65^\circ)} \right) - \frac{1}{2} g \left( \frac{5}{(v_0 \cos 65^\circ)} \right)^2$$
PROBLEM 11.109 (Continued)

\[ v_0^2 = \frac{\frac{1}{2} (9.81)(25)}{\cos^2 65^\circ (5 \tan 65^\circ - y_{B,C})} \]

\[ = \frac{686.566}{5 \tan 65^\circ - y_{B,C}} \]

At Point B:
\[ v_0^2 = \frac{686.566}{5 \tan 65^\circ - 5} \quad \text{or} \quad (v_0)_B = 10.95 \text{ m/s} \]

At Point C:
\[ v_0^2 = \frac{686.566}{5 \tan 65^\circ - 5.9} \quad \text{or} \quad (v_0)_C = 11.93 \text{ m/s} \]

\[ 10.95 \text{ m/s} \leq v_0 \leq 11.93 \text{ m/s} \]
**PROBLEM 11.110**

A ball is dropped onto a step at Point A and rebounds with a velocity \( v_0 \) at an angle of 15° with the vertical. Determine the value of \( v_0 \), knowing that just before the ball bounces at Point B, its velocity \( v_B \) forms an angle of 12° with the vertical.

**SOLUTION**

First note

- \((v_x)_0 = v_0 \sin 15°\)
- \((v_y)_0 = v_0 \cos 15°\)

**Horizontal motion.** (Uniform)

- \(v_x = (v_x)_0 = v_0 \sin 15°\)

**Vertical motion.** (Uniformly accelerated motion)

- \(v_y = (v_y)_0 - gt\)
- \(y = 0 + (v_y)_0 t - \frac{1}{2} gt^2\)
- \(v_y = v_0 \cos 15° - gt = (v_0 \cos 15°) t - \frac{1}{2} gt^2\)

At Point B, \( v_y < 0 \)

Then

- \(\tan 12° = \frac{(v_x)_B}{(v_y)_B} = \frac{v_0 \sin 15°}{v_0 \cos 15° - gt_B}\)

or

- \(t_B = \frac{v_0 (\sin 15° + \cos 15°)}{g \tan 12°} = 0.22259 v_0\)

Noting that \(y_B = 0.2 \text{ m}\),

We have

- \(-0.2 = (v_0 \cos 15°)(0.22259 v_0) - \frac{1}{2} (9.81)(0.22259 v_0)^2\)

or

- \(v_0 = 2.67 \text{ m/s}\)
PROBLEM 11.111

A model rocket is launched from Point A with an initial velocity $v_0$ of 75 m/s. If the rocket's descent parachute does not deploy and the rocket lands 120 m from A, determine (a) the angle $\alpha$ that $v_0$ forms with the vertical, (b) the maximum height above Point A reached by the rocket, and (c) the duration of the flight.

SOLUTION

Set the origin at Point A.

Horizontal motion:

$$x = v_0 t \sin \alpha \quad \sin \alpha = \frac{x}{v_0 t} \quad (1)$$

Vertical motion:

$$y = v_0 t \cos \alpha - \frac{1}{2} gt^2$$

$$\cos \alpha = \frac{1}{v_0 t} \left( y + \frac{1}{2} gt^2 \right)$$

$$\sin^2 \alpha + \cos^2 \alpha = \frac{1}{(v_0 t)^2} \left[ x^2 + \left( \frac{y + \frac{1}{2} gt^2}{v_0 t} \right)^2 \right] = 1$$

$$x^2 + y^2 + gy^2 + \frac{1}{4} g^2 t^4 = v_0^2 t^2$$

$$\frac{1}{4} g^2 t^4 - \left( v_0^2 - gy \right) t^2 + (x^2 + y^2) = 0 \quad (3)$$

At Point B,

$$\sqrt{x^2 + y^2} = 120 \text{ m}, \quad x = 120 \cos 30^\circ \text{ m}$$

$$y = -120 \sin 30^\circ = -60 \text{ m}$$

$$\frac{1}{4} (9.81)^2 t^4 - [75^2 - (9.81)(-60)] t^2 + 120^2 = 0$$

$$24.059 t^4 - 6213.6 t^2 + 14400 = 0$$

$$t^2 = 255.926 \text{ s}^2 \quad \text{and} \quad 2.33367 \text{ s}^2$$

$$t = 15.998 \text{ s} \quad \text{and} \quad 1.5293 \text{ s}$$
PROBLEM 11.111 (Continued)

Restrictions on $\alpha$:

\[
0 \leq \alpha \leq 120^\circ
\]

\[
\tan \alpha = \frac{x}{y + \frac{1}{2} gt^2} = \frac{120 \cos 30^\circ}{-60 + (4.905)(15.998)^2} = 0.08694
\]

\[
\alpha = 4.9687^\circ
\]

and

\[
\frac{120 \cos 30^\circ}{-60 + (4.905)(1.5293)^2} = -2.14149
\]

\[
\alpha = 115.031^\circ
\]

Use $\alpha = 4.9687^\circ$ corresponding to the steeper possible trajectory.

(a) **Angle $\alpha$**

$\alpha = 4.97^\circ \nLeftarrow$

(b) **Maximum height**

$v_y = 0$ at $y = y_{\text{max}}$

$v_y = v_0 \cos \alpha - gt = 0$

\[
t = \frac{v_0 \cos \alpha}{g}
\]

\[
y_{\text{max}} = v_0 t \cos \alpha - \frac{1}{2} gt = \frac{v_0^2 \cos^2 \alpha}{2g}
\]

\[
= \frac{(75)^2 \cos^2 4.9687^\circ}{2(9.81)}
\]

$y_{\text{max}} = 285 \text{ m} \nLeftarrow$

(c) **Duration of the flight**

(time to reach B)

\[
t = 16.00 \text{ s} \nLeftarrow
PROBLEM 11.112

The initial velocity $v_0$ of a hockey puck is 160 km/h. Determine (a) the largest value (less than 45°) of the angle $\alpha$ for which the puck will enter the net, (b) the corresponding time required for the puck to reach the net.

SOLUTION

First note

$$v_0 = 160 \text{ km/h} = \frac{400}{9} \text{ m/s}$$

and

$$(v_x)_0 = v_0 \cos \alpha = \left(\frac{400}{9}\right) \cos \alpha$$

$$(v_y)_0 = v_0 \sin \alpha = \left(\frac{400}{9}\right) \sin \alpha$$

(a) **Horizontal motion.** (Uniform)

$$x = 0 + (v_x)_0 t = \left(\frac{400}{9} \cos \alpha\right) t$$

At the front of the net, $x = 5 \text{ m}$

Then

$$5 = \left(\frac{400}{9} \cos \alpha\right) t$$

or

$$t_{\text{enter}} = \frac{9}{80 \cos \alpha}$$

**Vertical motion.** (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2} gt^2$$

$$= \left(\frac{400}{9} \sin \alpha\right) t - \frac{1}{2} \left(9.81 \text{ m/s}^2\right) t^2$$

At the front of the net,

$$y_{\text{front}} = \left(\frac{400}{9} \sin \alpha\right) t_{\text{enter}} - \frac{1}{2} \left(9.81 \text{ m/s}^2\right) t_{\text{enter}}^2$$

$$= \left(\frac{400}{9} \sin \alpha\right) \left(\frac{9}{80 \cos \alpha}\right) - \frac{1}{2} \left(\frac{9}{80 \cos \alpha}\right)^2$$

$$= 5 \tan \alpha - \frac{81g}{12800 \cos^2 \alpha}$$

Now

$$\frac{1}{\cos^2 \alpha} = 1 + \tan^2 \alpha$$
**PROBLEM 11.112 (Continued)**

Then

\[ y_{\text{front}} = 5 \tan \alpha - \frac{81g}{12800} (1 + \tan^2 \alpha) \]

or

\[ \tan^2 \alpha - \frac{64000}{81g} \tan \alpha + \left( 1 + \frac{12800}{81g} y_{\text{front}} \right) = 0 \]

Then

\[ \tan \alpha = \frac{\frac{64000}{81g} \pm \left( \frac{64000}{81g} \right)^2 - 4 \left( 1 + \frac{12800}{81g} y_{\text{front}} \right)}{2} \]

or

\[ \tan \alpha = \frac{\frac{64000}{162 \times 9.81} \pm \left( \frac{64000}{162 \times 9.81} \right)^2 - \left( 1 + \frac{12800}{81 \times 9.81} y_{\text{front}} \right)}{2} \]

or

\[ \tan \alpha = 40.2713 \pm \left( (40.2713)^2 - (1 + 16.1085 y_{\text{front}}) \right)^{1/2} \]

Now \( 0 \leq y_{\text{front}} \leq 1.2 \) m so that the positive root will yield values of \( \alpha > 45^\circ \) for all values of \( y_{\text{front}} \).

When the negative root is selected, \( \alpha \) increases as \( y_{\text{front}} \) is increased. Therefore, for \( \alpha_{\text{max}} \), set

\[ y_{\text{front}} = y_C = 1.2 \text{ m} \]

Then

\[ \tan \alpha = 40.2713 - \left( (40.2713)^2 - (1 + 16.1085 + 1.2) \right)^{1/2} \]

or

\[ \alpha_{\text{max}} = 14.2093^\circ \]

\[ \alpha_{\text{max}} = 14.66^\circ \]

(b) We had found

\[ t_{\text{enter}} = \frac{9}{80 \cos \alpha} \]

or

\[ t_{\text{enter}} = \frac{9}{80 \cos 14.2093^\circ} \]

or

\[ t_{\text{enter}} = 0.1161 \text{ s} \]
PROBLEM 11.113

The pitcher in a softball game throws a ball with an initial velocity \( \mathbf{v}_0 \) of 72 km/h at an angle \( \alpha \) with the horizontal. If the height of the ball at Point B is 0.68 m, determine (a) the angle \( \alpha \), (b) the angle \( \theta \) that the velocity of the ball at Point B forms with the horizontal.

SOLUTION

First note

\[
v_0 = 72 \text{ km/h} = 20 \text{ m/s}
\]

and

\[
\begin{align*}
(\mathbf{v}_x)_0 &= v_0 \cos \alpha = (20 \text{ m/s}) \cos \alpha \\
(\mathbf{v}_y)_0 &= v_0 \sin \alpha = (20 \text{ m/s}) \sin \alpha
\end{align*}
\]

(a) **Horizontal motion.** (Uniform)

\[
x = 0 + (\mathbf{v}_x)_0 t = (20 \cos \alpha) t
\]

At Point B:

\[
14 = (20 \cos \alpha) t \quad \text{or} \quad t_B = \frac{7}{10 \cos \alpha}
\]

**Vertical motion.** (Uniformly accelerated motion)

\[
y = 0 + (\mathbf{v}_y)_0 t - \frac{1}{2} gt^2 = (20 \sin \alpha) t - \frac{1}{2} gt^2 \quad (g = 9.81 \text{ m/s}^2)
\]

At Point B:

\[
0.08 = (20 \sin \alpha) t_B - \frac{1}{2} g t_B^2
\]

Substituting for \( t_B \)

\[
0.08 = (20 \sin \alpha) \left( \frac{7}{10 \cos \alpha} \right) - \frac{1}{2} \left( \frac{7}{10 \cos \alpha} \right)^2
\]

or

\[
8 = 1400 \tan \alpha - \frac{1}{2} \frac{49}{\cos^2 \alpha}
\]

Now

\[
\frac{1}{\cos^2 \alpha} = \sec^2 \alpha = 1 + \tan^2 \alpha
\]
PROBLEM 11.113 (Continued)

Then

\[ 8 = 1400 \tan \alpha - 24.5g(1 + \tan^2 \alpha) \]

or

\[ 240.345 \tan^2 \alpha - 1400 \tan \alpha + 248.345 = 0 \]

Solving \( \alpha = 10.3786^\circ \) and \( \alpha = 79.949^\circ \)

Rejecting the second root because it is not physically reasonable, we have

\[ \alpha = 10.38^\circ \]

(b) We have

\[ v_x = (v_x)_0 = 20 \cos \alpha \]

and

\[ v_y = (v_y)_0 - gt = 20 \sin \alpha - gt \]

At Point B:

\[ (v_y)_B = 20 \sin \alpha - gt_B \]

\[ = 20 \sin \alpha - \frac{7g}{10 \cos \alpha} \]

Noting that at Point B, \( v_y < 0 \), we have

\[ \tan \theta = \frac{|(v_y)_B|}{v_x} \]

\[ = \frac{\frac{7g}{10 \cos \alpha} - 20 \sin \alpha}{20 \cos \alpha} \]

\[ = \frac{\frac{7}{200} 0.981 - \sin 10.3786^\circ}{\cos 10.3786^\circ} \]

or

\[ \theta = 9.74^\circ \]

\[ \theta = 9.74^\circ \]
PROBLEM 11.114*

A mountain climber plans to jump from A to B over a crevasse. Determine the smallest value of the climber’s initial velocity \( v_0 \) and the corresponding value of angle \( \alpha \) so that he lands at B.

**SOLUTION**

First note \[ (v_x)_0 = v_0 \cos \alpha \]
\[ (v_y)_0 = v_0 \sin \alpha \]

**Horizontal motion. (Uniform)**

\[ x = 0 + (v_x)_0 t = (v_0 \cos \alpha) t \]
At Point B:
\[ 1.8 = (v_0 \cos \alpha) t \]
or
\[ t_B = \frac{1.8}{v_0 \cos \alpha} \]

**Vertical motion. (Uniformly accelerated motion)**

\[ y = 0 + (v_y)_0 t - \frac{1}{2} gt^2 \]
\[ = (v_0 \sin \alpha) t - \frac{1}{2} gt^2 \quad (g = 9.81 \text{ m/s}^2) \]

At Point B:
\[ -1.4 = (v_0 \sin \alpha) t_B - \frac{1}{2} gt_B^2 \]
Substituting for \( t_B \)
\[ -1.4 = (v_0 \sin \alpha) \left( \frac{1.8}{v_0 \cos \alpha} \right) - \frac{1}{2} g \left( \frac{1.8}{v_0 \cos \alpha} \right)^2 \]
or
\[ v_0^2 = \frac{1.62g}{\cos^2 \alpha (1.8 \tan \alpha + 1.4)} \]
\[ = \frac{1.62g}{0.9 \sin 2\alpha + 1.4 \cos^2 \alpha} \]
PROBLEM 11.114* (Continued)

Now minimize \( v_0^2 \) with respect to \( \alpha \).

We have

\[
\frac{dv_0^2}{d\alpha} = 1.62g \frac{-(1.8 \cos 2\alpha - 2.8 \cos \alpha \sin \alpha)}{(0.9 \sin 2\alpha + 1.4 \cos^2 \alpha)^2} = 0
\]

or

\[1.8 \cos 2\alpha - 1.4 \sin 2\alpha = 0\]

or

\[\tan 2\alpha = \frac{18}{14}\]

or

\[\alpha = 26.0625^\circ \quad \text{and} \quad \alpha = 206.06^\circ\]

Rejecting the second value because it is not physically possible, we have

\[\alpha = 26.1^\circ \quad \blacktriangleleft\]

Finally,

\[v_0^2 = \frac{1.62 \times 9.81}{\cos^2 26.0625^\circ (1.8 \tan 26.0625^\circ + 1.4)}\]

or

\[(v_0)_{\min} = 2.94 \text{ m/s} \quad \blacktriangleleft\]
**PROBLEM 11.115**

An oscillating garden sprinkler which discharges water with an initial velocity $v_0$ of 8 m/s is used to water a vegetable garden. Determine the distance $d$ to the farthest Point B that will be watered and the corresponding angle $\alpha$ when (a) the vegetables are just beginning to grow, (b) the height $h$ of the corn is 1.8 m.

**SOLUTION**

First note

$$v_x = v_0 \cos \alpha = (8 \text{ m/s}) \cos \alpha$$
$$v_y = v_0 \sin \alpha = (8 \text{ m/s}) \sin \alpha$$

**Horizontal motion.** (Uniform)

$$x = 0 + (v_x)_0 t = (8 \cos \alpha) t$$

At Point B:

$$x = d: \quad d = (8 \cos \alpha) t$$

or

$$t_B = \frac{d}{8 \cos \alpha}$$

**Vertical motion.** (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2} gt^2$$

$$= (8 \sin \alpha) t - \frac{1}{2} gt^2 \quad (g = 9.81 \text{ m/s}^2)$$

At Point B:

$$0 = (8 \sin \alpha) t_B - \frac{1}{2} gt_B^2$$

Simplifying and substituting for $t_B$

$$0 = 8 \sin \alpha - \frac{1}{2} g \left( \frac{d}{8 \cos \alpha} \right)^2$$

or

$$d = \frac{64}{9.81} \sin 2\alpha$$

(1)

(a) When $h = 0$, the water can follow any physically possible trajectory. It then follows from Eq. (1) that $d$ is maximum when $2\alpha = 90^\circ$

or

$$\alpha = 45^\circ$$

Then

$$d = \frac{64}{9.81} \sin (2 \times 45^\circ)$$

or

$$d_{\text{max}} = 6.52 \text{ m}$$
PROBLEM 11.115 (Continued)

(b) Based on Eq. (1) and the results of Part a, it can be concluded that \( d \) increases in value as \( \alpha \) increases in value from 0 to 45° and then \( d \) decreases as \( \alpha \) is further increased. Thus, \( d_{\text{max}} \) occurs for the value of \( \alpha \) closest to 45° and for which the water just passes over the first row of corn plants. At this row, \( x_{\text{com}} = 1.5 \text{ m} \) so that

\[
t_{\text{com}} = \frac{1.5}{8 \cos \alpha}
\]

Also, with \( y_{\text{corn}} = h \), we have

\[
h = (8 \sin \alpha) t_{\text{com}} - \frac{1}{2} g t_{\text{com}}^2
\]

Substituting for \( t_{\text{com}} \) and noting \( h = 1.8 \text{ m} \),

\[
1.8 = (8 \sin \alpha) \left( \frac{1.5}{8 \cos \alpha} \right) - \frac{1}{2} \left( \frac{1.5}{8 \cos \alpha} \right)^2
\]

or

\[
1.8 = \frac{1.5 \tan \alpha - \frac{2.25g}{128 \cos^2 \alpha}}{1}
\]

Now

\[
\frac{1}{\cos^2 \alpha} = \sec^2 \alpha = 1 + \tan^2 \alpha
\]

Then

\[
1.8 = \frac{1.5 \tan \alpha - \frac{2.25(9.81)}{128}(1 + \tan^2 \alpha)}{1}
\]

or

\[
0.172441 \tan^2 \alpha - 1.5 \tan \alpha + 1.972441 = 0
\]

Solving

\[
\alpha = 58.229^\circ \quad \text{and} \quad \alpha = 81.965^\circ
\]

From the above discussion, it follows that \( d = d_{\text{max}} \) when

\[
\alpha = 58.2^\circ \quad \blacktriangle
\]

Finally, using Eq. (1)

\[
d = \frac{64}{9.81} \sin(2 \times 58.229^\circ)
\]

or

\[
d_{\text{max}} = 5.84 \text{ m} \quad \blacktriangle
\]
**PROBLEM 11.116**

A worker uses high-pressure water to clean the inside of a long drainpipe. If the water is discharged with an initial velocity \( v_0 \) of 11.5 m/s, determine (a) the distance \( d \) to the farthest Point B on the top of the pipe that the worker can wash from his position at A, (b) the corresponding angle \( \alpha \).

**SOLUTION**

First note

\[
(v_x)_0 = v_0 \cos \alpha = (11.5 \text{ m/s}) \cos \alpha \\
(v_y)_0 = v_0 \sin \alpha = (11.5 \text{ m/s}) \sin \alpha
\]

By observation, \( d_{\text{max}} \) occurs when \( y_{\text{max}} = 1.1 \text{ m} \)

**Vertical motion.** (Uniformly accelerated motion)

\[
v_y = (v_y)_0 - gt \\
y = 0 + (v_y)_0 t - \frac{1}{2} gt^2 \\
= (11.5 \sin \alpha) - gt = (11.5 \sin \alpha) t - \frac{1}{2} gt^2
\]

When

\[
y = y_{\text{max}} \text{ at } B, \quad (v_y)_B = 0
\]

Then

\[
(v_y)_B = 0 = (11.5 \sin \alpha) - gt
\]

or

\[
t_B = \frac{11.5 \sin \alpha}{g} \quad (g = 9.81 \text{ m/s}^2)
\]

and

\[
y_B = (11.5 \sin \alpha) t_B - \frac{1}{2} g t_B^2
\]

Substituting for \( t_B \) and noting \( y_B = 1.1 \text{ m} \)

\[
1.1 = (11.5 \sin \alpha) \left( \frac{11.5 \sin \alpha}{g} \right) - \frac{1}{2} g \left( \frac{11.5 \sin \alpha}{g} \right)^2
\]

\[
\frac{1}{2g} (11.5)^2 \sin^2 \alpha
\]

or

\[
\sin^2 \alpha = \frac{2.2 \times 9.81}{11.5^2} \quad \alpha = 23.8265^\circ
\]
PROBLEM 11.116 (Continued)

(a) Horizontal motion. (Uniform)

\[ x = 0 + (v_x)_0 t = (11.5 \cos \alpha) t \]

At Point B:

\[ x = d_{\text{max}} \quad \text{and} \quad t = t_B \]

where

\[ t_B = \frac{11.5}{9.81} \sin 23.8265^\circ = 0.47356 \text{ s} \]

Then

\[ d_{\text{max}} = (11.5)(\cos 23.8265^\circ)(0.47356) \]

or

\[ d_{\text{max}} = 4.98 \text{ m} \]

(b) From above

\[ \alpha = 23.8^\circ \]

\[ d_{\text{max}} = 4.98 \text{ m} \]
**PROBLEM 11.117**

As slider block $A$ moves downward at a speed of 0.5 m/s, the velocity with respect to $A$ of the portion of belt $B$ between idler pulleys $C$ and $D$ is $v_{CD/A} = 2 \text{ m/s} \theta$. Determine the velocity of portion $CD$ of the belt when (a) $\theta = 45^\circ$, (b) $\theta = 60^\circ$.

**SOLUTION**

We have

$$v_{CD} = v_A + v_{CD/A}$$

where

$$v_A = (0.5 \text{ m/s})(-\cos 65^\circ \hat{i} - \sin 65^\circ \hat{j})$$

$$= (0.21131 \text{ m/s})\hat{i} - (0.45315 \text{ m/s})\hat{j}$$

and

$$v_{CD/A} = (2 \text{ m/s})(\cos \theta \hat{i} + \sin \theta \hat{j})$$

Then

$$v_{CD} = [(-0.21131 + 2 \cos \theta) \text{ m/s}]\hat{i} + [(-0.45315 + 2 \sin \theta) \text{ m/s}]\hat{j}$$

(a) We have

$$v_{CD} = (-0.21131 + 2 \cos 45^\circ)\hat{i} + (-0.45315 + 2 \sin 45^\circ)\hat{j}$$

$$= (1.20290 \text{ m/s})\hat{i} + (0.96106 \text{ m/s})\hat{j}$$

or

$$v_{CD} = 1.540 \text{ m/s} \angle 38.6^\circ$$

(b) We have

$$v_{CD} = (-0.21131 + 2 \cos 60^\circ)\hat{i} + (-0.45315 + 2 \sin 60^\circ)\hat{j}$$

$$= (0.78869 \text{ m/s})\hat{i} + (1.27890 \text{ m/s})\hat{j}$$

or

$$v_{CD} = 1.503 \text{ m/s} \angle 38.3^\circ$$
PROBLEM 11.118

The velocities of skiers A and B are as shown. Determine the velocity of A with respect to B.

SOLUTION

We have
\[ \mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{AB} \]

The graphical representation of this equation is then as shown.

Then
\[ \mathbf{v}_{AB}^2 = 10^2 + 14^2 - 2(10)(14) \cos 15^\circ \]

or
\[ \mathbf{v}_{AB} = 5.05379 \text{ m/s} \]

and
\[ \frac{10}{\sin \alpha} = \frac{5.05379}{\sin 15^\circ} \]

or
\[ \alpha = 30.8^\circ \]

\[ \mathbf{v}_{AB} = 5.05 \text{ m/s} \hat{\theta} 55.8^\circ \]

Alternative solution.

\[ \mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B \]

\[ = 10 \cos 10^\circ \mathbf{i} - 10 \sin 10^\circ \mathbf{j} - (14 \cos 25^\circ \mathbf{i} - 14 \sin 25^\circ \mathbf{j}) \]

\[ = -2.84 \mathbf{i} + 4.14 \mathbf{j} \]

\[ = 5.05 \text{ m/s} \hat{\theta} 55.8^\circ \]
PROBLEM 11.119

Shore-based radar indicates that a ferry leaves its slip with a velocity \( \mathbf{v} = 9.8 \text{ knots } \vec{\theta} 70^\circ \), while instruments aboard the ferry indicate a speed of 10 knots and a heading of 30° west of south relative to the river. Determine the velocity of the river.

SOLUTION

We have

\[
\mathbf{v}_F = \mathbf{v}_R + \mathbf{v}_{F/R} \quad \text{or} \quad \mathbf{v}_F = \mathbf{v}_{F/R} + \mathbf{v}_R
\]

The graphical representation of the second equation is then as shown.

We have

\[
\mathbf{v}_R^2 = 9.8^2 + 10^2 - 2(9.8)(10) \cos 10^\circ
\]

or

\[
\mathbf{v}_R = 1.737147 \text{ knots}
\]

and

\[
\frac{9.8}{\sin \alpha} = \frac{1.737147}{\sin 10^\circ}
\]

or

\[
\alpha = 78.41^\circ
\]

Noting that

\[
\mathbf{v}_R = 1.737 \text{ knots } \vec{\theta} 18.41^\circ
\]

\[= 3.22 \text{ km/h}\]

Alternatively one could use vector algebra.
PROBLEM 11.120

Airplanes A and B are flying at the same altitude and are tracking the eye of hurricane C. The relative velocity of C with respect to A is \( \mathbf{v}_{C/A} = 350 \text{ km/h} \) at \( 75^\circ \), and the relative velocity of C with respect to B is \( \mathbf{v}_{C/B} = 390 \text{ km/h} \) at \( 40^\circ \). Determine (a) the relative velocity of B with respect to A, (b) the velocity of A if ground-based radar indicates that the hurricane is moving at a speed of 36 km/h due north, (c) the change in position of C with respect to B during a 15-min interval.

SOLUTION

(a) We have
\[ \mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A} \]
and
\[ \mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B} \]
Then
\[ \mathbf{v}_A + \mathbf{v}_{C/A} = \mathbf{v}_B + \mathbf{v}_{C/B} \]
or
\[ \mathbf{v}_B - \mathbf{v}_A = \mathbf{v}_{C/A} - \mathbf{v}_{C/B} \]
Now
\[ \mathbf{v}_B - \mathbf{v}_A = \mathbf{v}_{B/A} \]
so that
\[ \mathbf{v}_{B/A} = \mathbf{v}_{C/A} - \mathbf{v}_{C/B} \]
or
\[ \mathbf{v}_{C/A} = \mathbf{v}_{C/B} + \mathbf{v}_{B/A} \]
The graphical representation of the last equation is then as shown.

We have
\[ v_{B/A}^2 = 350^2 + 390^2 - 2(350)(390) \cos 65^\circ \]
or
\[ v_{B/A} = 399.0303 \text{ km/h} \]
and
\[ 390 = \frac{399.0303}{\sin \alpha} \]
or
\[ \alpha = 62.3498^\circ \]
\[ v_{B/A} = 399 \text{ km/h} \] at \( 12.65^\circ \).

(b) We have
\[ \mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A} \]
or
\[ \mathbf{v}_A = (36 \text{ km/h}) \mathbf{j} - (350 \text{ km/h})(-\cos 75^\circ \mathbf{i} - \sin 75^\circ \mathbf{j}) \]
\[ \mathbf{v}_A = (90.587 \text{ km/h}) \mathbf{i} + (374.074 \text{ km/h}) \mathbf{j} \]
or
\[ \mathbf{v}_A = 385 \text{ km/h} \] at \( 76.4^\circ \).
PROBLEM 11.120 (Continued)

c) Noting that the velocities of B and C are constant. We have

\[ \mathbf{r}_B = (\mathbf{r}_B)_0 + \mathbf{v}_B t \quad \mathbf{r}_C = (\mathbf{r}_C)_0 + \mathbf{v}_C t \]

Now

\[ \mathbf{r}_{C/B} = \mathbf{r}_C - \mathbf{r}_B = [(\mathbf{r}_C)_0 - (\mathbf{r}_B)_0] + (\mathbf{v}_C - \mathbf{v}_B)t \]

Then

\[ \Delta \mathbf{r}_{C/B} = (\mathbf{r}_{C/B})_{t_2} - (\mathbf{r}_{C/B})_{t_1} = \mathbf{v}_{C/B} (t_2 - t_1) = \mathbf{v}_{C/B} \Delta t \]

For \( \Delta t = 15 \text{ min} \):

\[ \Delta \mathbf{r}_{C/B} = (390 \text{ km/h}) \left( \frac{1}{4} \text{ h} \right) = 97.5 \text{ km} \]

\[ \Delta \mathbf{r}_{C/B} = 97.5 \text{ km} \angle 40^\circ \]
PROBLEM 11.121

The velocities of commuter trains A and B are as shown. Knowing that the speed of each train is constant and that B reaches the crossing 10 min after A passed through the same crossing, determine (a) the relative velocity of B with respect to A, (b) the distance between the fronts of the engines 3 min after A passed through the crossing.

SOLUTION

(a) We have \[ \mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \]

The graphical representation of this equation is then as shown.

Then \[ v_{B/A}^2 = 66^2 + 48^2 - 2(66)(48) \cos 155^\circ \]

or \[ v_{B/A} = 111.366 \text{ km/h} \]

and \[ 48 = \frac{111.366}{\sin 155^\circ} \]

or \[ \alpha = 10.50^\circ \]

\[ v_{B/A} = 111.4 \text{ km/h} \angle 10.50^\circ \]

(b) First note that

at \( t = 3 \text{ min} \), A is \( (66 \text{ km/h})(3) = 3.3 \text{ km} \) west of the crossing.

at \( t = 3 \text{ min} \), B is \( (48 \text{ km/h})(7) = 5.6 \text{ km} \) southwest of the crossing.

Now \[ \mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A} \]

Then at \( t = 3 \text{ min} \), we have \[ r_{B/A}^2 = 3.3^2 + 5.6^2 - 2(3.3)(5.6) \cos 25^\circ \]

or \[ r_{B/A} = 2.96 \text{ km} \]
PROBLEM 11.122

Knowing that the velocity of block B with respect to block A is \( \mathbf{v}_{B/A} = 5.6 \text{ m/s} \atop 70^\circ \), determine the velocities of A and B.

SOLUTION

From the diagram

\[ 2x_A + 3x_B = \text{constant} \]

Then

\[ 2v_A + 3v_B = 0 \]

or

\[ |v_B| = \frac{2}{3} v_A \]

Now

\[ \mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \]

and noting that \( \mathbf{v}_A \) and \( \mathbf{v}_B \) must be parallel to surfaces A and B, respectively, the graphical representation of this equation is then as shown. Note: Assuming that \( \mathbf{v}_A \) is directed up the incline leads to a velocity diagram that does not “close.”

First note

\[ \alpha = 180^\circ - (40^\circ + 30^\circ + \theta_B) \]

\[ = 110^\circ - \theta_B \]

Then

\[ \frac{v_A}{\sin (110^\circ - \theta_B)} = \frac{\frac{2}{3} v_A}{\sin 40^\circ} = \frac{5.6}{\sin (30^\circ + \theta_B)} \]

or

\[ v_A \sin 40^\circ = \frac{2}{3} v_A \sin (110^\circ - \theta_B) \]

or

\[ \sin (110^\circ - \theta_B) = 0.96418 \]

or

\[ \theta_B = 35.3817^\circ \]

and

\[ \theta_B = 4.6183^\circ \]

For \( \theta_B = 35.3817^\circ \):

\[ v_B = \frac{2}{3} v_A = \frac{5.6 \sin 40^\circ}{\sin (30^\circ + 35.3817^\circ)} \]

or

\[ v_A = 5.94 \text{ m/s} \quad v_B = 3.96 \text{ m/s} \]

\[ \mathbf{v}_A = 5.94 \text{ m/s} \atop 30^\circ \]

\[ \mathbf{v}_B = 3.96 \text{ m/s} \atop 35.4^\circ \]
PROBLEM 11.122 (Continued)

For \( \theta_B = 4.6183^\circ \):

\[
\begin{align*}
V_B &= \frac{2}{3} V_A = \frac{5.6 \sin 40^\circ}{\sin (30^\circ + 4.6183^\circ)} \\
\text{or} & \\
V_A &= 9.50 \text{ m/s} \\
V_B &= 6.34 \text{ m/s}
\end{align*}
\]

\( V_A = 9.50 \text{ m/s} \quad \rightarrow \quad 30^\circ \quad \blacktriangleleft \)

\( V_B = 6.34 \text{ m/s} \quad \rightarrow \quad 4.62^\circ \quad \blacktriangleleft \)
PROBLEM 11.123

Knowing that at the instant shown block A has a velocity of 200 mm/s and an acceleration of 150 mm/s² both directed down the incline, determine (a) the velocity of block B, (b) the acceleration of block B.

SOLUTION

From the diagram

\[ 2x_A + x_{B/A} = \text{constant} \]

Then

\[ 2v_A + v_{B/A} = 0 \]

or

\[ |v_{B/A}| = 400 \text{ mm/s} \]

and

\[ 2a_A + a_{B/A} = 0 \]

or

\[ |a_{B/A}| = 300 \text{ mm/s}^2 \]

Note that \( v_{B/A} \) and \( a_{B/A} \) must be parallel to the top surface of block A.

(a) We have

\[ v_B = v_A + v_{B/A} \]

The graphical representation of this equation is then as shown. Note that because A is moving downward, B must be moving upward relative to A.

We have

\[ v_B^2 = 200^2 + 400^2 - 2(200)(400)\cos 15^\circ \]

or

\[ v_B = 213.194 \text{ mm/s} \]

and

\[ \frac{200}{\sin \alpha} = \frac{213.194}{\sin 15^\circ} \]

or

\[ \alpha = 14.05^\circ \]

\[ v_B = 213.194 \text{ mm/s} \]

(b) The same technique that was used to determine \( v_B \) can be used to determine \( a_B \). An alternative method is as follows.

We have

\[ a_B = a_A + a_{B/A} \]

\[ = (150\hat{i}) + 300(-\cos 15^\circ\hat{i} + \sin 15^\circ\hat{j}) \]

or

\[ a_B = 159.9 \text{ mm/s}^2 \]

* Note the orientation of the coordinate axes on the sketch of the system.
**PROBLEM 11.124**

Knowing that at the instant shown assembly A has a velocity of 225 mm/s and an acceleration of 375 mm/s² both directed downward, determine (a) the velocity of block B, (b) the acceleration of block B.

**SOLUTION**

Length of cable = constant

\[ L = x_A + 2x_{B/A} = \text{constant} \]

\[ v_A + 2v_{B/A} = 0 \]  
(1)

\[ a_A + 2a_{B/A} = 0 \]  
(2)

Data:

- \( a_A = 375 \text{ mm/s}^2 \)
- \( v_A = 225 \text{ mm/s} \)

Eqs. (1) and (2)

- \( a_A = -2a_{B/A} \)
- \( v_A = -2v_{B/A} \)
- \( 375 = -2a_{B/A} \)
- \( 225 = -2v_{B/A} \)
- \( a_{B/A} = -187.5 \text{ mm/s}^2 \)
- \( v_{B/A} = -112.5 \text{ mm/s} \)
- \( v_{B/A} = -112.5 \text{ mm/s} \)

(a) **Velocity of B**.

**Law of cosines:**

\[ v_B^2 = (225)^2 + (112.5)^2 - 2(225)(112.5)\cos 50^\circ \]

\[ v_B = 175.329 \text{ mm/s} \]

**Law of sines:**

\[ \frac{\sin \beta}{112.5} = \frac{\sin 50^\circ}{175.329} \]

\[ \beta = 29.44^\circ \]

\[ \alpha = 90^\circ - \beta = 90^\circ - 29.44^\circ = 60.56^\circ \]

\[ v_B = 175.3 \text{ mm/s} \]

\[ \beta = 29.44^\circ \]

\[ \alpha = 60.6^\circ \]
PROBLEM 11.124 (Continued)

(b) Acceleration of \( B \). \( \mathbf{a}_B \) may be found by using analysis similar to that used above for \( \mathbf{v}_B \). An alternate method is

\[
\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}
\]

\[
\mathbf{a}_B = 375 \text{ mm/s}^2 \downarrow + 187.5 \text{ mm/s}^2 \nearrow 40^\circ
\]

\[
= -375 \mathbf{j} - (187.5 \cos 40^\circ) \mathbf{i} + (187.5 \sin 40^\circ) \mathbf{j}
\]

\[
= -375 \mathbf{j} - 143.633 \mathbf{i} + 120.523 \mathbf{j}
\]

\[
\mathbf{a}_B = -143.633 \mathbf{i} - 254.477 \mathbf{j}
\]

\[
\mathbf{a}_B = 292 \text{ mm/s}^2 \nearrow 60.6^\circ
\]
**PROBLEM 11.125**

The assembly of rod A and wedge B starts from rest and moves to the right with a constant acceleration of 2 mm/s². Determine (a) the acceleration of wedge C, (b) the velocity of wedge C when t = 10 s.

**SOLUTION**

(a) We have

\[ a_C = a_B + a_{CB} \]

The graphical representation of this equation is then as shown.

First note

\[ \alpha = 180° - (20° + 105°) \]

= 55°

Then

\[ \frac{a_C}{\sin 20°} = \frac{2}{\sin 55°} \]

\[ a_C = 0.83506 \text{ mm/s}^2 \]

(b) For uniformly accelerated motion

\[ v_C = 0 + a_C t \]

At t = 10 s:

\[ v_C = (0.83506 \text{ mm/s}^2)(10 \text{ s}) \]

= 8.3506 mm/s

or

\[ v_C = 8.35 \text{ mm/s} \]

\[ 75° \]

\[ \triangle \]
PROBLEM 11.126

As the truck shown begins to back up with a constant acceleration of 1.2 m/s², the outer section B of its boom starts to retract with a constant acceleration of 0.5 m/s² relative to the truck. Determine (a) the acceleration of section B, (b) the velocity of section B when t = 2 s.

SOLUTION

(a) We have

\[ a_B = a_A + a_{B/A} \]

The graphical representation of this equation is then as shown.

We have

\[ a_B^2 = 1.2^2 + 0.5^2 - 2(1.2)(0.5) \cos 50° \]

or

\[ a_B = 0.95846 \text{ m/s}^2 \]

and

\[ \frac{0.5}{\sin \alpha} = \frac{0.95846}{\sin 50°} \]

or

\[ \alpha = 23.6° \]

\[ a_B = 0.958 \text{ m/s}^2 \triangleleft 23.6° \]

(b) For uniformly accelerated motion

\[ v_B = 0 + a_B \cdot t \]

At \( t = 2 \) s:

\[ v_B = (0.95846 \text{ m/s}^2)(2 \text{ s}) \]

\[ = 1.91692 \text{ m/s} \]

or

\[ v_B = 1.917 \text{ m/s} \triangleleft 23.6° \]
PROBLEM 11.127

Conveyor belt $A$, which forms a $20^\circ$ angle with the horizontal, moves at a constant speed of $1.2 \text{ m/s}$ and is used to load an airplane. Knowing that a worker tosses duffel bag $B$ with an initial velocity of $0.75 \text{ m/s}$ at an angle of $30^\circ$ with the horizontal, determine the velocity of the bag relative to the belt as it lands on the belt.

SOLUTION

First determine the velocity of the bag as it lands on the belt. Now

$[(v_B)_x]_0 = (v_B)_0 \cos 30^\circ$

$= (0.75 \text{ m/s}) \cos 30^\circ$

$[(v_B)_y]_0 = (v_B)_0 \sin 30^\circ$

$= (0.75 \text{ m/s}) \sin 30^\circ$

**Horizontal motion. (Uniform)**

$x = 0 + [(v_B)_x]_0 t$

$(v_B)_x = [(v_B)_x]_0$

$= 0.75 \cos 30^\circ$

**Vertical motion. (Uniformly accelerated motion)**

$y = y_0 + [(v_B)_y]_0 t - \frac{1}{2} gt^2$

$(v_B)_y = [(v_B)_y]_0 - gt$

$= 0.45 + (0.75 \sin 30^\circ) t - \frac{1}{2} gt^2$

$= 0.75 \sin 30^\circ - gt$

The equation of the line collinear with the top surface of the belt is

$y = x \tan 20^\circ$

Thus, when the bag reaches the belt

$0.45 + (0.75 \sin 30^\circ) t - \frac{1}{2} gt^2 = [(0.75 \cos 30^\circ) t] \tan 20^\circ$

or

$\frac{1}{2} (9.81)t^2 + 0.75(\cos 30^\circ \tan 20^\circ - \sin 30^\circ) t - 0.45 = 0$

or

$4.905t^2 - 0.13859t - 0.45 = 0$

Solving

$t = 0.31735 \text{ s and } t = -0.28909 \text{ s (Reject)}$
PROBLEM 11.127 (Continued)

The velocity $\mathbf{v}_B$ of the bag as it lands on the belt is then

$$\mathbf{v}_B = (0.75 \cos 30^\circ)\mathbf{i} + [0.75 \sin 30^\circ - 9.81(0.31735)]\mathbf{j}$$

$$= (0.64952 \text{ m/s})\mathbf{i} - (2.7382 \text{ m/s})\mathbf{j}$$

Finally

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

or

$$\mathbf{v}_{B/A} = (0.64952\mathbf{i} - 2.7382\mathbf{j}) - 1.2(\cos 20^\circ\mathbf{i} + \sin 20^\circ\mathbf{j})$$

$$= -(0.47811 \text{ m/s})\mathbf{i} - (3.1486 \text{ m/s})\mathbf{j}$$

or

$$\mathbf{v}_{B/A} = 3.18 \text{ m/s} \left(\begin{array}{c} 81.4^\circ \end{array}\right)$$
PROBLEM 11.128

Determine the required velocity of the belt $B$ if the relative velocity with which the sand hits belt $B$ is to be (a) vertical, (b) as small as possible.

SOLUTION

A grain of sand will undergo projectile motion.

$\mathbf{v}_{s} = \mathbf{v}_{sb} = \text{constant} = -2.5 \text{ m/s}$

$y$-direction.

$\mathbf{v}_{sy} = \sqrt{2gh} = \sqrt{(2)(9.81 \text{ m/s}^2)(1.5 \text{ m})} = 5.4249 \text{ m/s}$

Relative velocity.

$\mathbf{v}_{s/B} = \mathbf{v}_{s} - \mathbf{v}_{B}$ (1)

(a) If $\mathbf{v}_{s/B}$ is vertical,

$-\mathbf{v}_{s/B} = -2.5\mathbf{i} - 5.4249\mathbf{j} = -\mathbf{v}_{B} \cos 15^\circ \mathbf{i} + \mathbf{v}_{B} \sin 15^\circ \mathbf{j}$

Equate components.

$i$: $0 = -2.5 + \mathbf{v}_{B} \cos 15^\circ$ $\mathbf{v}_{B} = \frac{2.5}{\cos 15^\circ} = 2.5882 \text{ m/s}$

$\mathbf{v}_{B} = 2.59 \text{ m/s} \uparrow \leftarrow 15^\circ$

(b) $\mathbf{v}_{s/C}$ is as small as possible, so make $\mathbf{v}_{s/B} \perp \mathbf{v}_{B}$ into (1).

$-\mathbf{v}_{s/B} \sin 15^\circ \mathbf{i} - \mathbf{v}_{s/B} \cos 15^\circ \mathbf{j} = -2.5\mathbf{i} - 5.4249\mathbf{j} + \mathbf{v}_{B} \cos 15^\circ \mathbf{i} - \mathbf{v}_{B} \sin 15^\circ \mathbf{j}$

Equate components and transpose terms.

$(\sin 15^\circ) \mathbf{v}_{s/B} + (\cos 15^\circ) \mathbf{v}_{B} = 2.5$

$(\cos 15^\circ) \mathbf{v}_{s/B} - (\sin 15^\circ) \mathbf{v}_{B} = 5.4249$

Solving,

$\mathbf{v}_{s/B} = 5.8871 \text{ m/s}$ $\mathbf{v}_{B} = 1.0107 \text{ m/s}$

$\mathbf{v}_{B} = 1.011 \text{ m/s} \uparrow \leftarrow 15^\circ$
**PROBLEM 11.129**

As observed from a ship moving due east at 9 km/h, the wind appears to blow from the south. After the ship has changed course and speed, and as it is moving due north at 6 km/h, the wind appears to blow from the southwest. Assuming that the wind velocity is constant during the period of observation, determine the magnitude and direction of the true wind velocity.

**SOLUTION**

\[
\mathbf{V}_{\text{wind}} = \mathbf{V}_{\text{ship}} + \mathbf{V}_{\text{wind/ship}}
\]

**Case 1**

\[
\mathbf{V}_{\text{w}} = \mathbf{V}_{s} + \mathbf{V}_{\text{w/s}}
\]

\[
\mathbf{V}_{s} = 9 \text{ km/h} \rightarrow; \quad \mathbf{V}_{\text{w/s}} \uparrow
\]

**Case 2**

\[
\mathbf{V}_{s} = 6 \text{ km/h} \uparrow; \quad \mathbf{V}_{\text{w/s}} \rightarrow
\]

\[
\tan \alpha = \frac{15}{9} = 1.6667
\]

\[
\alpha = 59.0^\circ
\]

\[
\mathbf{V}_{w} = \sqrt{9^2 + 15^2} = 17.49 \text{ km/h}
\]

\[
\mathbf{V}_{w} = 17.49 \text{ km/h} \angle 59.0^\circ
\]
PROBLEM 11.130

When a small boat travels north at 5 km/h, a flag mounted on its stern forms an angle $\theta = 50^\circ$ with the centerline of the boat as shown. A short time later, when the boat travels east at 20 km/h, angle $\theta$ is again 50°. Determine the speed and the direction of the wind.

SOLUTION

We have

$$v_W = v_B + v_{W/B}$$

Using this equation, the two cases are then graphically represented as shown.

With $v_W$ now defined, the above diagram is redrawn for the two cases for clarity.

Noting that

$$\theta = 180^\circ - (50^\circ + 90^\circ + \alpha)$$
$$\phi = 180^\circ - (50^\circ + \alpha)$$
$$= 40^\circ - \alpha$$
$$= 130^\circ - \alpha$$

We have

$$\frac{v_W}{\sin 50^\circ} = \frac{5}{\sin (40^\circ - \alpha)}$$
$$\frac{v_W}{\sin 50^\circ} = \frac{20}{\sin (130^\circ - \alpha)}$$
PROBLEM 11.130 (Continued)

Therefore

\[
\frac{5}{\sin(40^\circ - \alpha)} = \frac{20}{\sin(130^\circ - \alpha)}
\]

or

\[
\sin 130^\circ \cos \alpha - \cos 130^\circ \sin \alpha = 4(\sin 40^\circ \cos \alpha - \cos 40^\circ \sin \alpha)
\]

or

\[
\tan \alpha = \frac{\sin 130^\circ - 4 \sin 40^\circ}{\cos 130^\circ - 4 \cos 40^\circ}
\]

or

\[
\alpha = 25.964^\circ
\]

Then

\[
v_W = \frac{5 \sin 50^\circ}{\sin (40^\circ - 25.964^\circ)} = 15.79 \text{ km/h}
\]

\[
v_W = 15.79 \text{ km/h} \quad \angle 26.0^\circ
\]
PROBLEM 11.131

As part of a department store display, a model train D runs on a slight incline between the store’s up and down escalators. When the train and shoppers pass Point A, the train appears to a shopper on the up escalator B to move downward at an angle of 22° with the horizontal, and to a shopper on the down escalator C to move upward at an angle of 23° with the horizontal and to travel to the left. Knowing that the speed of the escalators is 1 m/s, determine the speed and the direction of the train.

SOLUTION

We have

\[ \mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B} \]
\[ \mathbf{v}_D = \mathbf{v}_C + \mathbf{v}_{D/C} \]

The graphical representations of these equations are then as shown.

Then

\[ \frac{\mathbf{v}_D}{\sin 8^\circ} = \frac{1}{\sin(22^\circ + \alpha)} \]
\[ \frac{\mathbf{v}_D}{\sin 7^\circ} = \frac{1}{\sin(23^\circ - \alpha)} \]

Equating the expressions for \( \frac{\mathbf{v}_D}{1} \)

\[ \sin 8^\circ = \sin 7^\circ \]
\[ \sin(22^\circ + \alpha) \]
\[ \sin(23^\circ - \alpha) \]

or

\[ \sin 8^\circ (\sin 23^\circ \cos \alpha - \cos 23^\circ \sin \alpha) = \sin 7^\circ (\sin 22^\circ \cos \alpha + \cos 22^\circ \sin \alpha) \]

or

\[ \tan \alpha = \frac{\sin 8^\circ \sin 23^\circ - \sin 7^\circ \sin 22^\circ}{\sin 8^\circ \cos 23^\circ + \sin 7^\circ \cos 22^\circ} \]

or

\[ \alpha = 2.0728^\circ \]

Then

\[ \mathbf{v}_D = \frac{(1) \sin 8^\circ}{\sin(22^\circ + 2.0728^\circ)} = 0.3412 \text{ m/s} \]

\[ \mathbf{v}_D = 0.341 \text{ m/s} \theta 2.07^\circ \]
PROBLEM 11.131 (Continued)

Alternate solution using components.

\[ \mathbf{v}_B = (1 \text{ m/s}) \cos 30^\circ = (0.866 \text{ m/s}) \mathbf{i} + (0.5 \text{ m/s}) \mathbf{j} \]

\[ \mathbf{v}_C = (1 \text{ m/s}) \cos 30^\circ = (0.866 \text{ m/s}) \mathbf{i} - (0.5 \text{ m/s}) \mathbf{j} \]

\[ \mathbf{v}_{D/B} = \mathbf{u}_1 \cos 22^\circ - \mathbf{u}_2 \sin 22^\circ = (u_1 \cos 22^\circ) \mathbf{i} - (u_2 \sin 22^\circ) \mathbf{j} \]

\[ \mathbf{v}_{D/C} = \mathbf{u}_2 \sin 23^\circ + (u_2 \cos 23^\circ) \mathbf{i} + (u_2 \sin 23^\circ) \mathbf{j} \]

\[ \mathbf{v}_D = \mathbf{v}_D = \mathbf{u} \cos \alpha = - (v_0 \cos \alpha) \mathbf{i} + (v_0 \sin \alpha) \mathbf{j} \]

\[ \mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B} = \mathbf{v}_C + \mathbf{v}_{D/C} \]

\[ 0.866 \mathbf{i} + 0.5 \mathbf{j} - (u_1 \cos 22^\circ) \mathbf{i} - (u_1 \sin 22^\circ) \mathbf{j} = 0.866 \mathbf{i} - 0.5 \mathbf{j} - (u_2 \cos 23^\circ) \mathbf{i} + (u_2 \sin 23^\circ) \mathbf{j} \]

Separate into components, and transpose and change

\[ u_1 \cos 22^\circ - u_2 \cos 23^\circ = 0 \]

\[ u_1 \sin 22^\circ + u_1 \sin 23^\circ = 1 \]

Solving for \( u_1 \) and \( u_2 \),

\[ u_1 = 1.3018 \text{ m/s} \quad u_2 = 1.3112 \text{ m/s} \]

\[ \mathbf{v}_D = 0.866 \mathbf{i} + 0.5 \mathbf{j} - (1.3018 \cos 22^\circ) \mathbf{i} - (1.3112 \sin 22^\circ) \mathbf{j} \]

\[ = -(0.341 \text{ m/s}) \mathbf{i} + (0.008158 \text{ m/s}) \mathbf{j} \]

\[ \mathbf{v}_D = 0.341 \text{ m/s} \cos 2.07^\circ \]
**PROBLEM 11.132**

The paths of raindrops during a storm appear to form an angle of 75° with the vertical and to be directed to the left when observed through a left-side window of an automobile traveling north at a speed of 60 km/h. When observed through a right-side window of an automobile traveling south at a speed of 45 km/h, the raindrops appear to form an angle of 60° with the vertical. If the driver of the automobile traveling north were to stop, at what angle and with what speed would she observe the drops to fall?

**SOLUTION**

We have

\[ \mathbf{v}_R = (\mathbf{v}_A)_1 + (\mathbf{v}_{RA})_1 \]

\[ \mathbf{v}_R = (\mathbf{v}_A)_2 + (\mathbf{v}_{RA})_2 \]

The graphical representations of these equations are then as shown. Note that the line of action of \((\mathbf{v}_{RA})_2\) must be directed as shown so that the second velocity diagram "closes."

From the diagram

\[ (v_R)_y = [60 + (v_R)_x] \tan 15^\circ \]

and

\[ (v_R)_y = [45 - (v_R)_x] \tan 30^\circ \]

Equating the expressions for \((v_R)_y\)

\[ [60 + (v_R)_x] \tan 15^\circ = [45 - (v_R)_x] \tan 30^\circ \]

or

\[ (v_R)_x = 11.7163 \text{ km/h} \]

Then

\[ (v_R)_y = (60 + 11.7163) \tan 15^\circ = 19.2163 \text{ km/h} \]

\[ v_R = 22.5 \text{ km/h} \quad \theta = 58.6^\circ \]
**PROBLEM 11.133**

Determine the peripheral speed of the centrifuge test cab A for which the normal component of the acceleration is 10 g.

**SOLUTION**

\[
a_n = 10g = 10(9.81 \text{ m/s}^2) = 98.1 \text{ m/s}^2
\]

\[
a_n = \frac{v^2}{\rho}
\]

\[
\rho = (8 \text{ m})(98.1 \text{ m/s}^2) = 784.8 \text{ m}^2/\text{s}^2
\]

\[
v = 28.0 \text{ m/s}
\]
**PROBLEM 11.134**

To test its performance, an automobile is driven around a circular test track of diameter $d$. Determine (a) the value of $d$ if when the speed of the automobile is 72 km/h, the normal component of the acceleration is $3.2 \text{ m/s}^2$, (b) the speed of the automobile if $d = 180 \text{ m}$ and the normal component of the acceleration is measured to be 0.6 g.

**SOLUTION**

(a) First note $v = 72 \text{ km/h} = 20 \text{ m/s}$

Now $a_n = \frac{v^2}{\rho}$

or $d = \frac{(20 \text{ m/s})^2}{3.2 \text{ m/s}^2}$

or $d = 250 \text{ m}$

(b) We have $a_n = \frac{v^2}{\rho}$

Then $v^2 = (0.6 \times 9.81 \text{ m/s}^2) \left( \frac{1}{2} \times 180 \text{ m} \right)$

or $v = 23.016 \text{ m/s}$

or $v = 82.9 \text{ km/h}$
**PROBLEM 11.135**

Determine the smallest radius that should be used for a highway if the normal component of the acceleration of a car traveling at 72 km/h is not to exceed 0.8 m/s².

**SOLUTION**

\[ a_n = \frac{v^2}{\rho} \]

\[ a_n = 0.8 \text{ m/s}^2 \]

\[ v = 72 \text{ km/h} = 20 \text{ m/s} \]

\[ 0.8 \text{ m/s}^2 = \frac{(20 \text{ m/s})^2}{\rho} \]

\[ \rho = 500 \text{ m} \]
PROBLEM 11.136

Determine the maximum speed that the cars of the roller-coaster can reach along the circular portion AB of the track if the normal component of their acceleration cannot exceed 3 g.

SOLUTION

We have

\[ a_n = \frac{v^2}{\rho} \]

Then

\[ (v_{\text{max}})_{AB}^2 = (3 \times 9.81 \text{ m/s}^2)(24 \text{ m}) \]

or

\[ (v_{\text{max}})_{AB} = 26.577 \text{ m/s} \]

or

\[ (v_{\text{max}})_{AB} = 95.7 \text{ km/h} \]
PROBLEM 11.137

Pin A, which is attached to link AB, is constrained to move in the circular slot CD. Knowing that at \( t = 0 \) the pin starts from rest and moves so that its speed increases at a constant rate of 20 mm/s\(^2\), determine the magnitude of its total acceleration when (a) \( t = 0 \), (b) \( t = 2 \) s.

SOLUTION

(a) At \( t = 0 \), \( v_A = 0 \), which implies \((a_A)_n = 0\)

\[
a_A = (a_A)t
\]

or

\[
a_A = 20 \text{ mm/s}^2
\]

(b) We have uniformly accelerated motion

\[
v_A = 0 + (a_A)_t \cdot t
\]

At \( t = 2 \) s:

\[
v_A = (20 \text{ mm/s}^2)(2 \text{ s}) = 40 \text{ mm/s}
\]

Now

\[
(a_A)_n = \frac{v_A^2}{P_A} = \frac{(40 \text{ mm/s})^2}{90 \text{ mm}} = 17.778 \text{ mm/s}^2
\]

Finally,

\[
a_A^2 = (a_A)_k^2 + (a_A)_n^2
\]

or

\[
a_A = 26.8 \text{ mm/s}^2
\]
PROBLEM 11.138

A monorail train starts from rest on a curve of radius 400 m and accelerates at the constant rate $a$. If the maximum total acceleration of the train must not exceed 1.5 m/s$^2$, determine (a) the shortest distance in which the train can reach a speed of 72 km/h, (b) the corresponding constant rate of acceleration $a$.

SOLUTION

When $v = 72$ km/h = 20 m/s and $r = 400$ m,

$$a_n = \frac{v^2}{\rho} = \frac{(20)^2}{400} = 1.000 \text{ m/s}^2$$

But

$$a = \sqrt{a_n^2 + a_t^2}$$

$$a_t = \sqrt{a^2 - a_n^2} = \sqrt{(1.5)^2 - (1.000)^2} = \pm 1.11803 \text{ m/s}^2$$

Since the train is accelerating, reject the negative value.

(a) **Distance to reach the speed.**

Let

$$v_0 = 0$$

$$x_0 = 0$$

$$v_1^2 = v_0^2 + 2a_t(x_1 - x_0) = 2a_t x_1$$

$$x_1 = \frac{v_1^2}{2a_t} = \frac{(20)^2}{2(1.11803)} \Rightarrow x_1 = 178.9 \text{ m}$$

(b) **Corresponding tangential acceleration.**

$$a_t = 1.118 \text{ m/s}^2$$
**PROBLEM 11.139**

An outdoor track is 125 m in diameter. A runner increases her speed at a constant rate from 4 to 7 m/s over a distance of 30 m. Determine the total acceleration of the runner 2 s after she begins to increase her speed.

**SOLUTION**

We have uniformly accelerated motion

\[ v^2 = v_1^2 + 2a_1s_{12} \]

Substituting

\[
(7 \text{ m/s})^2 = (4 \text{ m/s})^2 + 2a_1(30 \text{ m})
\]

or

\[ a_1 = 0.55 \text{ m/s}^2 \]

Also

\[ v = v_1 + at \]

At \( t = 2 \text{ s} \):

\[ v = 4 \text{ m/s} + (0.55 \text{ m/s}^2)(2 \text{ s}) = 5.1 \text{ m/s} \]

Now

\[ a_n = \frac{v^2}{\rho} \]

At \( t = 2 \text{ s} \):

\[ a_n = \frac{(5.1 \text{ m/s})^2}{62.5 \text{ m}} = 0.41616 \text{ m/s}^2 \]

Finally

\[ a^2 = a_1^2 + a_n^2 \]

At \( t = 2 \text{ s} \):

\[ a^2 = 0.55^2 + 0.41616^2 \]

or

\[ a = 0.690 \text{ m/s}^2 \]
PROBLEM 11.140

At a given instant in an airplane race, airplane A is flying horizontally in a straight line, and its speed is being increased at the rate of 8 m/s². Airplane B is flying at the same altitude as airplane A and, as it rounds a pylon, is following a circular path of 300-m radius. Knowing that at the given instant the speed of B is being decreased at the rate of 3 m/s², determine, for the positions shown, (a) the velocity of B relative to A, (b) the acceleration of B relative to A.

SOLUTION

First note

\[ v_A = 450 \text{ km/h} \quad v_B = 540 \text{ km/h} = 150 \text{ m/s} \]

(a) We have

\[ v_B = v_A + v_{B/A} \]

The graphical representation of this equation is then as shown.

We have

\[ v_{B/A}^2 = v_A^2 + v_{B/A}^2 - 2v_A v_{B/A} \cos 60° \]

or \[ v_{B/A} = 501.10 \text{ km/h} \]

and \[ \frac{540}{\sin \alpha} = \frac{501.10}{\sin 60°} \]

or \[ \alpha = 68.9° \]

(b) First note

\[ a_A = 8 \text{ m/s}^2 \rightarrow (a_B)_t = 3 \text{ m/s}^2 \quad \triangle 60° \]

Now

\[ (a_B)_n = \frac{v_{B/A}^2}{\rho_B} = \frac{(150 \text{ m/s})^2}{300 \text{ m}} \]

or \[ (a_B)_n = 75 \text{ m/s}^2 \quad \triangle 30° \]

Then

\[ a_B = (a_B)_t + (a_B)_n \]

\[ = 3(-\cos 60° \hat{i} + \sin 60° \hat{j}) + 75(-\cos 30° \hat{i} - \sin 30° \hat{j}) \]

\[ = -(66.452 \text{ m/s}^2 \hat{i} - 34.902 \text{ m/s}^2 \hat{j}) \]

Finally

\[ a_B = a_A + a_{B/A} \]

or \[ a_{B/A} = (-66.452 \hat{i} - 34.902 \hat{j}) - (8\hat{i}) \]

\[ = -(74.452 \text{ m/s}^2 \hat{i} - 34.902 \text{ m/s}^2 \hat{j}) \]

or \[ a_{B/A} = 82.2 \text{ m/s}^2 \quad \triangle 25.1° \]
**PROBLEM 11.141**

A motorist traveling along a straight portion of a highway is decreasing the speed of his automobile at a constant rate before exiting from the highway onto a circular exit ramp with a radius of 170 m. He continues to decelerate at the same constant rate so that 10 s after entering the ramp, his speed has decreased to 30 km/h, a speed which he then maintains. Knowing that at this constant speed the total acceleration of the automobile is equal to one quarter of its value prior to entering the ramp, determine the maximum value of the total acceleration of the automobile.

**SOLUTION**

First note

\[ v_{10} = 30 \text{ km/h} = \frac{25}{3} \text{ m/s} \]

While the car is on the straight portion of the highway,

\[ a = a_{\text{straight}} = a_t \]

and for the circular exit ramp

\[ a = \sqrt{a_t^2 + a_n^2} \]

where

\[ a_n = \frac{v^2}{\rho} \]

By observation, \( a_{\text{max}} \) occurs when \( v \) is maximum, which is at \( t = 0 \) when the car first enters the ramp.

For uniformly decelerated motion

\[ v = v_0 + a_t t \]

and at \( t = 10 \text{ s} \):

\[ v = \text{constant} \Rightarrow a = a_n = \frac{v_0^2}{\rho} \]

\[ a = \frac{1}{4} a_{\text{st}} \]

Then

\[ a_{\text{straight}} = a_t \Rightarrow \frac{1}{4} a_t = \frac{v_0^2}{\rho} = \frac{(\frac{25}{3} \text{ m/s})^2}{170 \text{ m}} \]

or

\[ a_t = -1.63399 \text{ m/s}^2 \]

(The car is decelerating; hence the minus sign.)
PROBLEM 11.141 (Continued)

Then at $t = 10$ s:

$$\frac{25}{3} \text{ m/s} = v_0 + (-1.63399 \text{ m/s}^2)(10 \text{ s})$$

or

$$v_0 = 24.6732 \text{ m/s}$$

Then at $t = 0$:

$$a_{\text{max}} = \sqrt{a^2 + \left(\frac{v_0}{\rho}\right)^2}$$

$$= \sqrt{(-1.63399 \text{ m/s}^2)^2 + \left(\frac{24.6732 \text{ m/s}}{170 \text{ m}}\right)^2}$$

or

$$a_{\text{max}} = 3.94 \text{ m/s}^2$$
**PROBLEM 11.142**

Racing cars A and B are traveling on circular portions of a race track. At the instant shown, the speed of A is decreasing at the rate of 7 m/s², and the speed of B is increasing at the rate of 2 m/s². For the positions shown, determine (a) the velocity of B relative to A, (b) the acceleration of B relative to A.

**SOLUTION**

First note

\[ v_A = 162 \text{ km/h} = 45 \text{ m/s} \]

\[ v_B = 144 \text{ km/h} = 40 \text{ m/s} \]

(a) We have

\[ v_B = v_A + v_{B/A} \]

The graphical representation of this equation is then as shown.

We have

\[ v_{B/A}^2 = v_A^2 + 144^2 - 2(162)(144) \cos 165° \]

or

\[ v_{B/A} = 303.39 \text{ km/h} \]

and

\[ \frac{144}{\sin \alpha} = \frac{303.39}{\sin 165°} \]

or

\[ \alpha = 7.056° \]

(b) First note

\[ (a_A)_t = 7 \text{ m/s}^2 \angle 60° \]

\[ (a_B)_t = 2 \text{ m/s}^2 \angle 45° \]

Now

\[ a_n = \frac{v^2}{\rho} \]

Then

\[ (a_A)_n = \frac{(45 \text{ m/s})^2}{300 \text{ m}} \quad (a_B)_n = \frac{(40 \text{ m/s})^2}{250 \text{ m}} \]

or

\[ (a_A)_n = 6.75 \text{ m/s}^2 \angle 30° \quad (a_B)_n = 6.40 \text{ m/s}^2 \angle 45° \]
PROBLEM 11.142 (Continued)

Noting that 

\[ \mathbf{a} = \mathbf{a}_i + \mathbf{a}_n \]

We have 

\[ \mathbf{a}_i = 7(\cos 60^\circ \mathbf{i} - \sin 60^\circ \mathbf{j}) + 6.75(-\cos 30^\circ \mathbf{i} - \sin 30^\circ \mathbf{j}) \]

\[ = -(2.3457 \text{ m/s}^2) \mathbf{i} - (9.4372 \text{ m/s}^2) \mathbf{j} \]

and 

\[ \mathbf{a}_n = 2(\cos 45^\circ \mathbf{i} - \sin 45^\circ \mathbf{j}) + 6.40(\cos 45^\circ \mathbf{i} + \sin 45^\circ \mathbf{j}) \]

\[ = (5.9397 \text{ m/s}^2) \mathbf{i} + (3.1113 \text{ m/s}^2) \mathbf{j} \]

Finally 

\[ \mathbf{a}_n = \mathbf{a}_i + \mathbf{a}_n \]

or 

\[ \mathbf{a}_{B/A} = (5.9397 \mathbf{i} + 3.1113 \mathbf{j}) - (2.3457 \mathbf{i} - 9.4372 \mathbf{j}) \]

\[ = (8.2854 \text{ m/s}^2) \mathbf{i} + (12.5485 \text{ m/s}^2) \mathbf{j} \]

or 

\[ \mathbf{a}_{B/A} = 15.04 \text{ m/s}^2 \angle 56.6^\circ \]
PROBLEM 11.143

A golfer hits a golf ball from Point A with an initial velocity of 50 m/s at an angle of 25° with the horizontal. Determine the radius of curvature of the trajectory described by the ball (a) at Point A, (b) at the highest point of the trajectory.

SOLUTION

(a) We have \( (a_n)_A = \frac{v_A^2}{\rho_A} \)

or \( \rho_A = \frac{(50 \text{ m/s})^2}{(9.81 \text{ m/s}^2) \cos 25°} \)

or \( \rho_A = 281 \text{ m} \)

(b) We have \( (a_n)_B = \frac{v_B^2}{\rho_B} \)

where Point B is the highest point of the trajectory, so that \( v_B = (v_A)_x = v_A \cos 25° \)

Then \( \rho_B = \frac{[(50 \text{ m/s}) \cos 25°]^2}{9.81 \text{ m/s}^2} \)

or \( \rho_B = 209 \text{ m} \)
PROBLEM 11.144

From a photograph of a homeowner using a snowblower, it is determined that the radius of curvature of the trajectory of the snow was 8.5 m as the snow left the discharge chute at A. Determine (a) the discharge velocity $v_A$ of the snow, (b) the radius of curvature of the trajectory at its maximum height.

SOLUTION

(a) We have

$$\alpha_A = \frac{v_A^2}{\rho_A}$$

or

$$v_A^2 = (9.81 \cos 40^\circ)(8.5 \text{ m})$$

$$= 63.8766 \text{ m}^2/\text{s}^2$$

or

(b) We have

$$\alpha_B = \frac{v_B^2}{\rho_B}$$

where Point B is the highest point of the trajectory, so that

$$v_B = (v_A)_{xB} = v_A \cos 40^\circ$$

Then

$$\rho_B = \frac{(63.8766 \text{ m}^2/\text{s}^2 \cos^2 40^\circ)}{9.81 \text{ m/s}^2}$$

or

$$\rho_B = 3.82 \text{ m}$$
PROBLEM 11.145

A basketball is bounced on the ground at Point A and rebounds with a velocity \( \mathbf{v}_A \) of magnitude 2 m/s as shown. Determine the radius of curvature of the trajectory described by the ball (a) at Point A, (b) at the highest point of the trajectory.

SOLUTION

(a) We have \((a_A)_n = \frac{\mathbf{v}_A^2}{\rho_A}\)

or \(\rho_A = \frac{(2 \text{ m/s})^2}{(9.81 \text{ m/s}^2) \sin 15^\circ} = 1.5754 \text{ m}\)

or \(\rho_A = 1.575 \text{ m}\)

(b) We have \((a_B)_n = \frac{\mathbf{v}_B^2}{\rho_B}\)

where Point B is the highest point of the trajectory, so that \(\mathbf{v}_B = (\mathbf{v}_A)_x = \mathbf{v}_A \sin 15^\circ\)

Then \(\rho_B = \frac{[(2 \text{ m/s}) \sin 15^\circ]^2}{9.81 \text{ m/s}^2} = 0.02713 \text{ m}\)

or \(\rho_B = 0.0271 \text{ m}\)
PROBLEM 11.146

Coal is discharged from the tailgate A of a dump truck with an initial velocity \( v_A = 2 \text{ m/s} \) at \( 50^\circ \). Determine the radius of curvature of the trajectory described by the coal (a) at Point A, (b) at the point of the trajectory 1 m below Point A.

SOLUTION

(a) We have

\[
(a_A)_n = \frac{v_A^2}{\rho_A}
\]

or

\[
\rho_A = \frac{(2 \text{ m/s})^2}{(9.81 \text{ m/s}^2) \cos 50^\circ}
\]

or

\[
\rho_A = 0.6341 \text{ m}
\]

(b) Horizontal motion. (Uniform)

\[
(v_B)_x = (v_A)_x = (2 \text{ m/s}) \cos 50^\circ = 1.28558 \text{ m/s}
\]

Vertical motion. (Uniformly accelerated motion)

We have

\[
v_y^2 = (v_A)_y^2 - 2g(y - y_A)
\]

where

\[
(v_A)_y = (2 \text{ m/s}) \sin 50^\circ = 1.53209 \text{ m/s}
\]

At Point B, \( y = -1 \text{ m} \):

\[
(v_B)_y = (1.53209 \text{ m/s})^2 - 2(9.81 \text{ m/s}^2)(-1 \text{ m})
\]

or

\[
(v_B)_y = 4.6869 \text{ m/s}
\]

Then

\[
v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = \sqrt{(1.28558)^2 + (4.6869)^2} = 4.8600 \text{ m/s}
\]

and

\[
\tan \theta = \frac{v_y}{v_x} = \frac{4.6869}{1.28558}
\]

or

\[
\theta = 74.661^\circ
\]

Now

\[
(a_B)_n = \frac{v_B}{\rho_B}
\]

or

\[
\rho_B = \frac{(4.86 \text{ m/s})^2}{(9.81 \text{ m/s}^2) \cos(74.661^\circ)}
\]

or

\[
\rho_B = 9.10 \text{ m}
\]
PROBLEM 11.147

A horizontal pipe discharges at Point A a stream of water into a reservoir. Express the radius of curvature of the stream at Point B in terms of the magnitudes of the velocities \( v_A \) and \( v_B \).

**SOLUTION**

We have

\[
(a_B)_n = \frac{v_B^2}{\rho_B}
\]

where

\[
(a_B)_n = a_B \cos \theta = g \cos \theta
\]

Noting that the horizontal motion is uniform, we have

\[
(v_B)_x = v_A
\]

where

\[
(v_B)_x = v_B \cos \theta
\]

\[
\cos \theta = \frac{v_A}{v_B}
\]

Then

\[
\rho_B = \frac{v_B^2}{g \left( \frac{v_A}{v_B} \right)}
\]

or

\[
\rho_B = \frac{v_B^2}{gv_A}
\]
PROBLEM 11.148

A child throws a ball from Point A with an initial velocity $v_A$ of 20 m/s at an angle of 25° with the horizontal. Determine the velocity of the ball at the points of the trajectory described by the ball where the radius of curvature is equal to three-quarters of its value at A.

SOLUTION

Assume that Point B and C are the points of interest, where $y_B = y_C$ and $v_B = v_C$.

Now

$$(a_B)_n = \frac{v_B^2}{\rho_B}$$

or

$$\rho_B = \frac{v_B^2}{g \cos 25°}$$

Then

$$\rho_B = \frac{3}{4} \rho_A = \frac{3}{4} \frac{v_A^2}{g \cos 25°}$$

We have

$$(a_B)_n = \frac{v_B^2}{\rho_B}$$

where

$$(a_B)_n = g \cos \theta$$

so that

$$\frac{3}{4} \frac{v_A^2}{g \cos 25°} = \frac{v_B^2}{g \cos \theta}$$

or

$$v_B^2 = \frac{3 \cos \theta}{4 \cos 25°} v_A^2$$

(1)

Noting that the horizontal motion is uniform, we have

$$(v_A)_x = (v_B)_x$$

where

$$(v_A)_x = v_A \cos 25° \quad (v_B)_x = v_B \cos \theta$$

Then

$$v_A \cos 25° = v_B \cos \theta$$

or

$$\cos \theta = \frac{v_A \cos 25°}{v_B}$$
PROBLEM 11.148 (Continued)

Substituting for $\cos \theta$ in Eq. (1), we have

$$v_B^2 = \frac{3}{4} \left( \frac{v_A}{v_B} \cos 25^\circ \right) - \frac{v_A^2}{\cos 25^\circ}$$

or

$$v_B^2 = \frac{3}{4} v_A^2$$

$$v_B = \sqrt[3]{\frac{3}{4}} v_A = 18.17 \text{ m/s}$$

$$\cos \theta = \frac{\sqrt[3]{3}}{4} \cos 25^\circ$$

$$\theta = \pm 4.04^\circ$$

and

$$v_B = 18.17 \text{ m/s} \angle 4.04^\circ$$

and

$$v_B = 18.17 \text{ m/s} \angle 4.04^\circ$$
PROBLEM 11.149

A projectile is fired from Point A with an initial velocity \( \mathbf{v}_0 \). (a) Show that the radius of curvature of the trajectory of the projectile reaches its minimum value at the highest Point B of the trajectory. (b) Denoting by \( \theta \) the angle formed by the trajectory and the horizontal at a given Point C, show that the radius of curvature of the trajectory at C is \( \rho = \rho_{\text{min}}/\cos^3 \theta \).

SOLUTION

For the arbitrary Point C, we have

\[
(a_C)_n = \frac{\mathbf{v}_C^2}{\rho_C}
\]

or

\[
\rho_C = \frac{\mathbf{v}_C^2}{g \cos \theta}
\]

Noting that the horizontal motion is uniform, we have

\[
(v_A)_x = (v_C)_x
\]

where

\[
(v_A)_x = v_0 \cos \alpha \quad (v_C)_x = v_C \cos \theta
\]

Then

\[
v_0 \cos \alpha = v_C \cos \theta
\]

or

\[
v_C = \frac{\cos \alpha}{\cos \theta} v_0
\]

so that

\[
\rho_C = \frac{1}{g \cos \theta} \left( \frac{\cos \alpha}{\cos \theta} v_0 \right)^2 = \frac{v_0^2 \cos^2 \alpha}{g \cos^3 \theta}
\]

(a) In the expression for \( \rho_C \), \( v_0, \alpha, \) and \( g \) are constants, so that \( \rho_C \) is minimum where \( \cos \theta \) is maximum. By observation, this occurs at Point B where \( \theta = 0 \).

\[
\rho_{\text{min}} = \rho_B = \frac{\mathbf{v}_0^2 \cos^2 \alpha}{g}
\]

Q.E.D.

(b) \[
\rho_C = \frac{1}{\cos^3 \theta} \left( \frac{\mathbf{v}_0^2 \cos^2 \alpha}{g} \right)
\]

\[
\rho_C = \frac{\rho_{\text{min}}}{\cos^3 \theta}
\]

Q.E.D.
PROBLEM 11.150

A projectile is fired from Point A with an initial velocity $v_0$ which forms an angle $\alpha$ with the horizontal. Express the radius of curvature of the trajectory of the projectile at Point C in terms of $x, v_0, \alpha$, and $g$.

SOLUTION

We have

$$\left( a_x \right)_n = \frac{\dot{v}_C^2}{\rho_C}$$

or

$$\rho_C = \frac{\dot{v}_C^2}{g \cos \theta}$$

Noting that the horizontal motion is uniform, we have

$$(v_A)_x = (v_C)_x \quad x = 0 + (v_0)_x t = (v_0 \cos \alpha) t$$

where

$$(v_A)_x = v_0 \cos \alpha \quad \quad (v_C)_x = v_C \cos \theta$$

Then

$$v_0 \cos \alpha = v_C \cos \theta \quad \quad \text{and} \quad (v_C)_x = v_0 \cos \alpha \quad (1)$$

or

$$\cos \theta = \frac{v_0 \cos \alpha}{v_C}$$

so that

$$\rho_C = \frac{\dot{v}_C^2}{g v_C \cos \alpha}$$

For the uniformly accelerated vertical motion have

$$(v_C)_y = (v_0)_y - gt = v_0 \sin \alpha - gt$$

From above

$$x = (v_0 \cos \alpha) t \quad \quad \text{or} \quad t = \frac{x}{v_0 \cos \alpha}$$

Then

$$(v_C)_y = v_0 \sin \alpha - g \frac{x}{v_0 \cos \alpha} \quad (2)$$

Now

$$\dot{v}_C^2 = (v_C)_x^2 + (v_C)_y^2$$
PROBLEM 11.150 (Continued)

Substituting for \((v_C)_x\) [Eq. (1)] and \((v_C)_y\) [Eq. (2)]

\[
v_C^2 = (v_0 \cos \alpha)^2 + \left( v_0 \sin \alpha - \frac{x}{v_0 \cos \alpha} \right)^2
\]

\[
= v_0^2 \left( 1 - \frac{2gx \tan \alpha}{v_0^2} + \frac{g^2x^2}{v_0^4 \cos^2 \alpha} \right)
\]

or

\[
v_C^3 = v_0^3 \left( 1 - \frac{2gx \tan \alpha}{v_0^2} + \frac{g^2x^2}{v_0^4 \cos^2 \alpha} \right)^{3/2}
\]

Finally, substituting into the expression for \(\rho_C\), obtain

\[
\rho = \frac{v_0^2}{g \cos \alpha} \left( 1 - \frac{2gx \tan \alpha}{v_0^2} + \frac{g^2x^2}{v_0^4 \cos^2 \alpha} \right)^{3/2}
\]
PROBLEM 11.151*

Determine the radius of curvature of the path described by the particle of Problem 11.95 when t = 0.

**SOLUTION**

We have

\[ \mathbf{v} = \frac{d\mathbf{r}}{dt} = R(\cos \omega_n t - \omega_n t \sin \omega_n t)\mathbf{i} + c\mathbf{j} + R(\sin \omega_n t + \omega_n t \cos \omega_n t)\mathbf{k} \]

and

\[ \mathbf{a} = \frac{d\mathbf{v}}{dt} = R(-\omega_n \sin \omega_n t - \omega_n \cos \omega_n t - \omega_n^2 t \cos \omega_n t)\mathbf{i} \]

\[ + R(\omega_n \cos \omega_n t + \omega_n \cos \omega_n t - \omega_n^2 t \sin \omega_n t)\mathbf{k} \]

or

\[ \mathbf{a} = \omega_n R[-(2 \sin \omega_n t + \omega_n t \cos \omega_n t)\mathbf{i} \]

\[ + (2 \cos \omega_n t - \omega_n t \sin \omega_n t)\mathbf{k}] \]

Now

\[ v^2 = R^2(\cos \omega_n t - \omega_n t \sin \omega_n t)^2 + c^2 \]

\[ + R^2(\sin \omega_n t + \omega_n t \cos \omega_n t)^2 \]

\[ = R^2(1 + \omega_n^2 t^2) + c^2 \]

Then

\[ v = \left[ R^2(1 + \omega_n^2 t^2) + c^2 \right]^{1/2} \]

and

\[ \frac{dv}{dt} = \frac{R^2 \omega_n t}{\left[ R^2(1 + \omega_n^2 t^2) + c^2 \right]^{1/2}} \]

Now

\[ a^2 = a_n^2 + a_n^2 \]

\[ = \left( \frac{dv}{dt} \right)^2 + \left( \frac{v}{\rho} \right)^2 \]

At t = 0:

\[ \frac{dv}{dt} = 0 \]

\[ a = \omega_n R(2\mathbf{k}) \quad \text{or} \quad a = 2\omega_n R \]

\[ v^2 = R^2 + c^2 \]

Then, with

\[ \frac{dv}{dt} = 0, \]

we have

\[ a = \frac{v^2}{\rho} \]

or

\[ 2\omega_n R = \frac{R^2 + c^2}{\rho} \]

or

\[ \rho = \frac{R^2 + c^2}{2\omega_n R} \]

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**PROBLEM 11.152**

Determine the radius of curvature of the path described by the particle of Problem 11.96 when \( t = 0, A = 3, \) and \( B = 1. \)

**SOLUTION**

With \( A = 3, \; B = 1 \),

We have

\[
\mathbf{r} = (3 \cos t) \mathbf{i} + \left( 3 \frac{t^2 + 1}{\sqrt{t^2 + 1}} \right) \mathbf{j} + (t \sin t) \mathbf{k}
\]

Now

\[
\mathbf{v} = \frac{d\mathbf{r}}{dt} = 3(\cos t - t \sin t) \mathbf{i} + \left( \frac{3t}{\sqrt{t^2 + 1}} \right) \mathbf{j} + (\sin t + t \cos t) \mathbf{k}
\]

and

\[
\mathbf{a} = \frac{d\mathbf{v}}{dt} = 3(-\sin t - \sin t - t \cos t) \mathbf{i} + 3 \left[ \frac{\sqrt{t^2 + 1} - t \left( \frac{t}{\sqrt{t^2 + 1}} \right)}{t^2 + 1} \right] \mathbf{j}
\]

\[
+ (\cos t + \cos t - t \sin t) \mathbf{k}
\]

\[
= -3(2 \sin t + t \cos t) \mathbf{i} + 3 \frac{1}{(t^2 + 1)^{1/2}} \mathbf{j}
\]

\[
+ (2 \cos t - t \sin t) \mathbf{k}
\]

Then

\[
\mathbf{v}^2 = 9(\cos t - t \sin t)^2 + 9 \frac{t^2}{t^2 + 1} + (\sin t + t \cos t)^2
\]

Expanding and simplifying yields

\[
\mathbf{v}^2 = t^4 + 19t^2 + 1 + 8(\cos^2 t + t^4 \sin^2 t) - 8(t^3 + t) \sin 2t
\]

Then

\[
\mathbf{v} = [t^4 + 19t^2 + 1 + 8(\cos^2 t + t^4 \sin^2 t) - 8(t^3 + t) \sin 2t]^{1/2}
\]

and

\[
\frac{d\mathbf{v}}{dt} = \frac{-4t^3 + 38t + 8(-2 \cos t \sin t + 4t^3 \sin^2 t + 2t^4 \sin^2 t) - 8[(3t^2 + 1) \sin 2t + 2(t^3 + t) \cos 2t]}{2[t^4 + 19t^2 + 1 + 8(\cos^2 t + t^4 \sin^2 t) - 8(t^3 + t) \sin 2t]^{1/2}}
\]

Now

\[
a^2 = a_x^2 + a_y^2 = \left( \frac{d\mathbf{v}}{dt} \right)^2 + \left( \frac{\mathbf{v}^2}{\rho} \right)^2
\]
PROBLEM 11.152* (Continued)

At $t = 0$:  

$$ \mathbf{a} = 3\mathbf{j} + 2\mathbf{k} $$  

or  

$$ a = \sqrt{13} \text{ m/s}^2 $$  

$$ \frac{dv}{dt} = 0 $$  

$$ v^2 = 9 \text{ (m/s)}^2 $$

Then, with  

$$ \frac{dv}{dt} = 0, $$

we have  

$$ a = \frac{v^2}{\rho} $$

or  

$$ \rho = \frac{9 \text{ m}^2/\text{s}^2}{\sqrt{13} \text{ m/s}^2} $$

or  

$$ \rho = 2.50 \text{ m} \blacktriangle $$
PROBLEM 11.153

A satellite will travel indefinitely in a circular orbit around a planet if the normal component of the acceleration of the satellite is equal to \( g \left( \frac{R}{r} \right)^2 \), where \( g \) is the acceleration of gravity at the surface of the planet, \( R \) is the radius of the planet, and \( r \) is the distance from the center of the planet to the satellite. Determine the speed of a satellite relative to the indicated planet if the satellite is to travel indefinitely in a circular orbit 160 km above the surface of the planet.

Venus: \( g = 8.53 \text{ m/s}^2, R = 6161 \text{ km} \).

**SOLUTION**

We have

\[
\begin{align*}
a_n &= g \left( \frac{R}{r} \right)^2 \\
a_n &= \frac{v^2}{r}
\end{align*}
\]

Then

\[
\begin{align*}
g \left( \frac{R}{r} \right)^2 &= \frac{v^2}{r} \\
\end{align*}
\]

or

\[
\begin{align*}
v_{\text{circ}} &= R \sqrt{\frac{g}{r}} \\
\text{where} \quad r &= R + h
\end{align*}
\]

For the given data

\[
\begin{align*}
v_{\text{circ}} &= 6161 \text{ km} \left( \frac{8.53 \text{ m/s}^2}{(6161 + 160) \times 10^3 \text{ m}} \right) \times \frac{3600 \text{ s}}{1 \text{ h}} \\
\end{align*}
\]

or

\[
v_{\text{circ}} = 25.8 \times 10^3 \text{ km/h}
\]
**PROBLEM 11.154**

A satellite will travel indefinitely in a circular orbit around a planet if the normal component of the acceleration of the satellite is equal to \( g \left( \frac{R}{r} \right)^2 \), where \( g \) is the acceleration of gravity at the surface of the planet, \( R \) is the radius of the planet, and \( r \) is the distance from the center of the planet to the satellite. Determine the speed of a satellite relative to the indicated planet if the satellite is to travel indefinitely in a circular orbit 160 km above the surface of the planet.

Mars: \( g = 3.83 \text{ m/s}^2 \), \( R = 3332 \text{ km} \).

**SOLUTION**

We have

\[
\begin{align*}
a_n &= g \frac{R^2}{r^2} \quad \text{and} \quad a_n = \frac{v^2}{r} \tag{11.23}\end{align*}
\]

Then

\[
g \frac{R^2}{r^2} = \frac{v^2}{r} \tag{11.24}
\]

or

\[
v_{\text{circ}} = R \sqrt{\frac{g}{r}} \quad \text{where} \quad r = R + h
\]

For the given data

\[
v_{\text{circ}} = 3332 \text{ km} \sqrt{\frac{3.83 \text{ m/s}^2}{(3332 + 160) \times 10^3 \text{ m}}} \times \frac{3600 \text{ s}}{1 \text{ h}}
\]

or

\[
v_{\text{circ}} = 12.56 \times 10^3 \text{ km/h}
\]
PROBLEM 11.155

A satellite will travel indefinitely in a circular orbit around a planet if the normal component of the acceleration of the satellite is equal to \( g \left( \frac{R}{r} \right)^2 \), where \( g \) is the acceleration of gravity at the surface of the planet, \( R \) is the radius of the planet, and \( r \) is the distance from the center of the planet to the satellite. Determine the speed of a satellite relative to the indicated planet if the satellite is to travel indefinitely in a circular orbit 160 km above the surface of the planet.

Jupiter: \( g = 26.0 \text{ m/s}^2 \), \( R = 69,893 \text{ km} \).

SOLUTION

We have

\[
\frac{a_n}{g} \frac{R^2}{r^2} = \frac{v^2}{r}
\]

Then

\[
\frac{g R^2}{r^2} = \frac{v^2}{r}
\]

or

\[
v_{\text{circ.}} = \frac{R}{\sqrt{r}} \frac{g}{r} \text{ where } r = R + h
\]

For the given data

\[
v_{\text{circ.}} = 69,893 \sqrt[2]{\frac{26.0 \text{ m/s}^2}{(69,893+160) \times 10^3 \text{ m}}} \times \frac{3600 \text{ s}}{1 \text{ h}}
\]

or

\[
v_{\text{circ.}} = 153.3 \times 10^3 \text{ km/h}
\]
**PROBLEM 11.156**

Knowing that the diameter of the sun is $1.391 \times 10^6$ km and that the acceleration of gravity at its surface is 274 m/s\(^2\), determine the radius of the orbit of the indicated planet around the sun assuming that the orbit is circular. (See information given in Problems 11.153–11.155.)

Earth: \((v_{\text{mean}})_{\text{orbit}} = 1.072 \times 10^5 \text{ km/h}\)

**SOLUTION**

We have

\[
a_n = g \frac{R^2}{r^2} \quad \text{and} \quad \frac{v^2}{r} = \frac{a_n}{g}.
\]

Then

\[
\frac{g R^2}{r^2} = \frac{v^2}{r} = \frac{a_n}{g}.
\]

or

\[
r = g \left( \frac{R}{v} \right)^2 \quad \text{where} \quad R = \frac{1}{2} d.
\]

For the given data:

\[
g = 274 \text{ m/s}^2 = 274 \times \frac{(3600)^2}{1000} \text{ km/h}^2 = 3551040 \text{ km/h}^2
\]

\[
r_{\text{earth}} = 3551040 \text{ km/h}^2 \left( \frac{\frac{1}{2} \times 1.391 \times 10^6 \text{ km}}{1.072 \times 10^5 \text{ km/h}} \right)^2
\]

or

\[
r_{\text{earth}} = 149.5 \times 10^6 \text{ km}
\]
PROBLEM 11.157

Knowing that the diameter of the sun is $1.391 \times 10^6$ km and that the acceleration of gravity at its surface is $274 \text{ m/s}^2$, determine the radius of the orbit of the indicated planet around the sun assuming that the orbit is circular. (See information given in Problems 11.153–11.155.)

Saturn: $(v_{\text{mean}})_{\text{orbit}} = 3.47 \times 10^4$ km/h

SOLUTION

We have

$$a_n = g \frac{R^2}{r^2} \quad \text{and} \quad a_n = \frac{v^2}{r}$$

Then

$$g \frac{R^2}{r^2} = \frac{v^2}{r}$$

or

$$r = g \left( \frac{R}{v} \right)^2 \quad \text{where} \quad R = \frac{1}{2} d$$

For the given data:

$$g = 274 \text{ m/s}^2 = 274 \times \frac{(3600)^2}{1000} \text{ km/h}^2 = 3551040 \text{ km/h}^2$$

$$r_{\text{saturn}} = 3551040 \text{ km/h}^2 \left( \frac{1}{2} \times \frac{1.391 \times 10^6 \text{ km}}{3.47 \times 10^4 \text{ km/h}} \right)^2$$

or

$$r_{\text{saturn}} = 1.427 \times 10^9 \text{ km}$$
**PROBLEM 11.158**

Knowing that the radius of the earth is 6370 km, determine the time of one orbit of the Hubble Space Telescope, knowing that the telescope travels in a circular orbit 590 km above the surface of the earth. (See information given in Problems 11.153–11.155.)

**SOLUTION**

We have

\[ a_h = \frac{g R^2}{r^2} \quad \text{and} \quad a_h = \frac{v^2}{r} \]

Then

\[ g \frac{R^2}{r^2} = \frac{v^2}{r} \]

or

\[ v = R \sqrt{\frac{g}{r}} \quad \text{where} \quad r = R + h \]

The circumference \( s \) of the circular orbit is equal to

\[ s = 2\pi r \]

Assuming that the speed of the telescope is constant, we have

\[ s = vt_{orbit} \]

Substituting for \( s \) and \( v \)

\[ 2\pi r = R \sqrt{\frac{g}{r}} t_{orbit} \]

or

\[ t_{orbit} = \frac{2\pi r}{R} \frac{r^2}{\sqrt{g}} \]

or

\[ t_{orbit} = \frac{2\pi [(6370 + 590) \text{ km}]^2}{6370 \text{ km} [9.81 \times 10^{-3} \text{ km/s}^2]^2} \times \frac{1 \text{ h}}{3600 \text{ s}} \]

or

\[ t_{orbit} = 1.606 \text{ h} \]

\[ \blacksquare \]
PROBLEM 11.159

A satellite is traveling in a circular orbit around Mars at an altitude of 270 km. After the altitude of the satellite is adjusted, it is found that the time of one orbit has increased by 10 percent. Knowing that the radius of Mars is 3370 km, determine the new altitude of the satellite. (See information given in Problems 11.153–11.155.)

SOLUTION

We have

\[ a_n = \frac{gR^2}{r^2} \quad \text{and} \quad a_n = \frac{\sqrt{v}}{r} \]

Then

\[ \frac{gR^2}{r^2} = \frac{v^2}{r} \]

or

\[ v = R \sqrt{\frac{g}{r}} \quad \text{where} \quad r = R + h \]

The circumference \( s \) of a circular orbit is equal to

\[ s = 2\pi r \]

Assuming that the speed of the satellite in each orbit is constant, we have

\[ s = v \times \text{orbit} \]

Substituting for \( s \) and \( v \)

\[ 2\pi r = R \sqrt{\frac{g}{r}} \times \text{orbit} \]

or

\[ t_{\text{orbit}} = \frac{2\pi R^{3/2}}{\sqrt{g}} \]

Then

\[ \left( t_{\text{orbit}} \right)_2 = 1.1 \left( t_{\text{orbit}} \right)_1 \]

or

\[ \frac{2\pi (R + h)^{3/2}}{\sqrt{g}} = 1.1 \left( \frac{2\pi (R + h)^{3/2}}{\sqrt{g}} \right) \]

or

\[ h = (1.1)^{2/3} (R + h) - R \]

\[ = (1.1)^{2/3} (3370 + 270) \text{ km} - (3370 \text{ km}) \]

\[ = 508.792 \text{ km} \]

or

\[ h_2 = 509 \text{ km} \]
PROBLEM 11.160

Satellites A and B are traveling in the same plane in circular orbits around the earth at altitudes of 180 and 300 km, respectively. If at \( t = 0 \) the satellites are aligned as shown and knowing that the radius of the earth is \( R = 6370 \) km, determine when the satellites will next be radially aligned. (See information given in Problems 11.153–11.155.)

SOLUTION

We have

\[ a_n = g \frac{R^2}{r^2} \quad \text{and} \quad a_n = \frac{v^2}{r} \]

Then

\[ g \frac{R^2}{r^2} = \frac{v^2}{r} \quad \text{or} \quad v = R \sqrt{\frac{g}{r}} \]

where

\[ r = R + h \]

The circumference \( s \) of a circular orbit is

\[ s = 2\pi r \]

Assuming that the speeds of the satellites are constant, we have

\[ s = vT \]

Substituting for \( s \) and \( v \)

\[ 2\pi r = R \sqrt{\frac{g}{r} T} \]

or

\[ T = \frac{2\pi r^{3/2}}{R \sqrt{g}} = \frac{2\pi (R+h)^{3/2}}{R \sqrt{g}} \]

Now

\[ h_B > h_A \Rightarrow (T)_B > (T)_A \]

Next let time \( T_C \) be the time at which the satellites are next radially aligned. Then, if in time \( T_C \) satellite B completes \( N \) orbits, satellite A must complete \( (N+1) \) orbits.

Thus,

\[ T_C = N(T)_B = (N+1)(T)_A \]

or

\[ N \left[ \frac{2\pi (R+h_b)^{3/2}}{R \sqrt{g}} \right] = (N+1) \left[ \frac{2\pi (R+h_b)^{3/2}}{R \sqrt{g}} \right] \]
PROBLEM 11.160 (Continued)

or

\[ N = \frac{(R + h_b)^{3/2}}{(R + h_b)^{3/2} - (R + h_b)^{3/2}} = \frac{1}{R} - 1 \]

\[ = \frac{1 - 1}{\left(\frac{6370 + 300}{6370 + 180}\right)^{3/2}} = 36.223 \text{ orbits} \]

Then

\[ T_c = N(T)_B = N \frac{2\pi (R + h_b)^{3/2}}{R \sqrt{g}} \]

\[ = 36.223 \frac{2\pi}{6370} \frac{[(6370 + 300) \text{ km}]^{3/2}}{(9.81 \text{ m/s}^2 \times \frac{1 \text{ km}}{1000 \text{ m}})^{3/2}} \times \frac{1 \text{ h}}{3600 \text{ s}} \]

or

\[ T_c = 54.6 \text{ h} \]
**PROBLEM 11.161**

The path of a particle $P$ is a limaçon. The motion of the particle is defined by the relations $r = b(2 + \cos \pi t)$ and $\theta = \pi t$, where $t$ and $\theta$ are expressed in seconds and radians, respectively. Determine (a) the velocity and the acceleration of the particle when $t = 2 \, \text{s}$, (b) the values of $\theta$ for which the magnitude of the velocity is maximum.

**SOLUTION**

We have

$$r = b(2 + \cos \pi t) \quad \theta = \pi t$$

Then

$$\dot{r} = -\pi b \sin \pi t \quad \dot{\theta} = \pi$$

and

$$\ddot{r} = -\pi^2 b \cos \pi t \quad \dot{\theta} = 0$$

Now

$$\mathbf{v} = \dot{r} \hat{e}_r + r \dot{\theta} \hat{e}_\theta = (-\pi b \sin \pi t) \hat{e}_r + \pi b(2 + \cos \pi t) \hat{e}_\theta$$

and

$$\mathbf{a} = (\ddot{r} - r \ddot{\theta}) \hat{e}_r + (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \hat{e}_\theta$$

$$= [-\pi^2 b \cos \pi t - \pi^2 b(2 + \cos \pi t)] \hat{e}_r + (0 - 2\pi^2 b \sin \pi t) \hat{e}_\theta$$

$$= -2\pi^2 b[1 + \cos \pi t] \hat{e}_r + (\sin \pi t) \hat{e}_\theta$$

(a) At $t = 2 \, \text{s}$:

$$\mathbf{v} = -(0) \hat{e}_r + \pi b(2 + 1) \hat{e}_\theta$$

or

$$\mathbf{v} = 3\pi b \hat{e}_\theta$$

or

$$\mathbf{a} = -2\pi^2 b[1 + \cos \pi t] \hat{e}_r + (0) \hat{e}_\theta$$

or

$$\mathbf{a} = -4\pi^2 b \hat{e}_\theta$$

(b) We have

$$\mathbf{v} = \pi b \sqrt{(-\sin \pi t)^2 + (2 + \cos \pi t)^2}$$

$$= \pi b \sqrt{5 + 4 \cos \pi t} \quad \theta = \pi t$$

$$= \pi b \sqrt{5 + 4 \cos \theta}$$

By observation,

$$\mathbf{v} = v_{\text{max}} \quad \text{when} \quad \cos \theta = 1$$

or

$$\theta = 2n \pi, \quad n = 0, 1, 2, \ldots$$
PROBLEM 11.162
The two-dimensional motion of a particle is defined by the relation \( r = 2b \cos \omega t \) and \( \theta = \omega t \) where \( b \) and \( \omega \) are constant. Determine (a) the velocity and acceleration of the particle at any instant, (b) the radius of curvature of its path. What conclusion can you draw regarding the path of the particle?

SOLUTION

\( r = 2b \cos \omega t \quad \theta = \omega t \)

\( \dot{r} = -2b \omega \sin \omega t \quad \dot{\theta} = \omega \)

\( \ddot{r} = -2b \omega^2 \cos \omega t \quad \ddot{\theta} = 0 \)

(a) Velocity.

\[ v_x = \dot{r} = -2b \omega \sin \omega t \quad v_y = r \dot{\theta} = 2b \omega \cos \omega t \]

\[ v^2 = v_x^2 + v_y^2 = (2b \omega)^2 [(-\sin \omega t)^2 + (\cos \omega t)^2] = (2b \omega)^2 \]

\( v = 2b \omega \ )

Acceleration.

\[ a_x = \ddot{r} - r \ddot{\theta} = -2b \omega^2 \cos \omega t - (2b \cos \omega t)(\omega)^2 \]

\[ = -4b \omega^2 \cos \omega t \]

\[ a_y = r \ddot{\theta} + 2r \dot{\theta} = (2b \cos \omega t)(0) + 2(-2b \omega \sin \omega t)(\omega) \]

\[ = -4b \omega^2 \sin \omega t \]

\[ a^2 = a_x^2 + a_y^2 = (-4b \omega^2)^2 (\cos^2 \omega t + \sin^2 \omega t) = (4b \omega^2)^2 \]

\( a = 4b \omega^2 \ )

(b) Since \( v = 2b \omega \) = constant, \( a_x = 0 \)

Thus:

\[ a_n = a = 4b \omega^2 \]

\[ a_n = \frac{v^2}{\rho} \quad \rho = \frac{v^2}{a_n} = \frac{(2b \omega)^2}{4b \omega^2} = b \quad \rho = b \]

For the path, \( \rho = \) constant.

Thus, path is a circle.
PROBLEM 11.163

The rotation of rod OA about O is defined by the relation \( \theta = \pi(4t^2 - 8t) \), where \( \theta \) and \( t \) are expressed in radians and seconds, respectively. Collar B slides along the rod so that its distance from O is \( r = (250 + 150 \sin \pi t) \), where \( r \) and \( t \) are expressed in mm and seconds, respectively. When \( t = 1 \) s, determine (a) the velocity of the collar, (b) the total acceleration of the collar, (c) the acceleration of the collar relative to the rod.

SOLUTION

We have

\[
\begin{align*}
        r &= 25(10 + 6 \sin \pi t) & \theta &= \pi(4t^2 - 8t) \\
       r &= 150\pi \cos \pi t & \dot{\theta} &= \theta\pi(t - 1) \\
      \text{and} & & \ddot{\theta} &= \theta \pi \\
\text{At } t = 1 \text{ s:} & & r &= 250 \text{ mm} \\
                  & & \dot{r} &= 150\pi \text{ mm/s} \\
                  & & \ddot{r} &= 0 \\
(\text{a}) \quad \text{We have} & & \mathbf{v}_B &= r \mathbf{e}_r + r \dot{\theta} \mathbf{e}_\theta \\
\text{so that} & & \mathbf{v}_B &= -(150\pi \text{ mm/s}) \mathbf{e}_\theta \\
(\text{b}) \quad \text{We have} & & \mathbf{a}_B &= (r - r \ddot{\theta}^2) \mathbf{e}_r + (r \dddot{\theta} + 2r \dot{\theta}) \mathbf{e}_\theta \\
\text{or} & & \mathbf{a}_B &= (250)(8\pi^3) \mathbf{e}_\theta \\
(\text{c}) \quad \text{We have} & & \mathbf{a}_{B/OA} &= \ddot{r} \\
\text{so that} & & \mathbf{a}_{B/OA} &= 0
**PROBLEM 11.164**

The oscillation of rod OA about O is defined by the relation \( \theta = \frac{2}{\pi} \sin \pi t \), where \( \theta \) and \( t \) are expressed in radians and seconds, respectively. Collar B slides along the rod so that its distance from O is \( r = \frac{625}{t+4} \) where \( r \) and \( t \) are expressed in mm and seconds, respectively. When \( t = 1 \) s, determine (a) the velocity of the collar, (b) the total acceleration of the collar, (c) the acceleration of the collar relative to the rod.

---

**SOLUTION**

We have

\[
\begin{align*}
 r &= \frac{625}{t+4} \\
 \theta &= \frac{2}{\pi} \sin \pi t
\end{align*}
\]

Then

\[
\begin{align*}
 \dot{r} &= -\frac{625}{(t+4)^2} \\
 \dot{\theta} &= 2 \cos \pi t
\end{align*}
\]

and

\[
\begin{align*}
 \ddot{r} &= \frac{1250}{(t+4)^3} \\
 \ddot{\theta} &= -2 \pi \sin \pi t
\end{align*}
\]

At \( t = 1 \) s

\[
\begin{align*}
 r &= 125 \text{ mm} \\
 \dot{r} &= -25 \text{ mm/s} \\
 \ddot{r} &= 10 \text{ mm/s}^2
\end{align*}
\]

(a) We have

\[
\begin{align*}
 \mathbf{v}_B &= r \mathbf{e}_r + r \dot{\theta} \mathbf{e}_\theta = (-1) \mathbf{e}_r + (5)(-2) \mathbf{e}_\theta \\
 &= -(25 \text{ mm/s}) \mathbf{e}_r - (250 \text{ mm/s}) \mathbf{e}_\theta
\end{align*}
\]

or

\[
\mathbf{v}_A = -(25 \text{ mm/s}) \mathbf{e}_r - (250 \text{ mm/s}) \mathbf{e}_\theta
\]

(b) We have

\[
\begin{align*}
 \mathbf{a}_B &= (r - r \dot{\theta}^2) \mathbf{e}_r + (r \ddot{\theta} + 2r \dot{\theta}) \mathbf{e}_\theta \\
 &= [10 - (125)(2)^2] \mathbf{e}_r + [0 + 2(25)(-2)] \mathbf{e}_\theta \\
 &= -(490 \text{ mm/s}^2) \mathbf{e}_r + (100 \text{ mm/s}^2) \mathbf{e}_\theta
\end{align*}
\]

or

\[
\mathbf{a}_B = -(490 \text{ mm/s}^2) \mathbf{e}_r + (100 \text{ mm/s}^2) \mathbf{e}_\theta
\]

(c) We have

\[
\mathbf{a}_{B/OA} = \ddot{r}
\]

so that

\[
\mathbf{a}_{B/OA} = (10 \text{ mm/s}^2) \mathbf{e}_r
\]
PROBLEM 11.165

The path of particle P is the ellipse defined by the relations \( r = 2 / (2 - \cos \pi t) \) and \( \theta = \pi t \), where \( r \) is expressed in meters, \( t \) is in seconds, and \( \theta \) is in radians. Determine the velocity and the acceleration of the particle when (a) \( t = 0 \), (b) \( t = 0.5 \) s.

SOLUTION

We have

\[
\begin{align*}
r &= \frac{2}{2 - \cos \pi t} \\
\theta &= \pi t
\end{align*}
\]

Then

\[
\begin{align*}
r &= \frac{-2\pi \sin \pi t}{(2 - \cos \pi t)^2} \\
\dot{\theta} &= \pi
\end{align*}
\]

and

\[
\begin{align*}
r &= -2\pi \frac{\pi \cos \pi t (2 - \cos \pi t) - \sin \pi t (2\pi \sin \pi t)}{(2 - \cos \pi t)^3} \\
\ddot{\theta} &= 0
\end{align*}
\]

\[
\begin{align*}
r &= -2\pi^2 \frac{2 \cos \pi t - 1 - \sin^2 \pi t}{(2 - \cos \pi t)^3}
\end{align*}
\]

(a) At \( t = 0 \):

\[
\begin{align*}
r &= 2 \text{ m} \\
\dot{r} &= 0 \\
\ddot{r} &= -2\pi^2 \text{ m/s}^2
\end{align*}
\]

Now

\[
\begin{align*}
v &= r \dot{e}_r + r \dot{\theta} \dot{e}_\theta = (2\pi) \dot{e}_\theta
\end{align*}
\]

or

\[
\begin{align*}
v &= (2\pi \text{ m/s}) \dot{e}_\theta
\end{align*}
\]

and

\[
\begin{align*}
a &= (\ddot{r} - \dot{r} \dot{\theta}^2) \dot{e}_r + (\ddot{\theta} + 2\dot{\theta} \dot{r}) \dot{e}_\theta
\end{align*}
\]

or

\[
\begin{align*}
a &= [-2\pi^2 - (2\pi)^2] \dot{e}_\theta
\end{align*}
\]

(b) At \( t = 0.5 \) s:

\[
\begin{align*}
r &= 1 \text{ m} \\
\theta &= \frac{\pi}{2} \text{ rad}
\end{align*}
\]

\[
\begin{align*}
r &= \frac{-2\pi}{(2)^2} = -\frac{\pi}{2} \text{ m/s} \\
\dot{\theta} &= \pi \text{ rad/s}
\end{align*}
\]

\[
\begin{align*}
\ddot{r} &= -2\pi^2 \frac{-1 - 1}{(2)^3} = \frac{\pi^2}{2} \text{ m/s}^2 \\
\ddot{\theta} &= 0
\end{align*}
\]
PROBLEM 11.165 (Continued)

Now
\[ \mathbf{v} = r \mathbf{e}_r + \dot{r} \mathbf{e}_r + \left( -\frac{\pi}{2} \right) \mathbf{e}_\theta + (\pi) \mathbf{e}_\phi \]

or
\[ \mathbf{v} = -\left( \frac{\pi}{2} \text{ m/s} \right) \mathbf{e}_r + (\pi \text{ m/s}) \mathbf{e}_\phi \]

and
\[ \mathbf{a} = (r - r \dot{\theta}^2) \mathbf{e}_r + (r \dot{\theta} + 2r \dot{\phi}) \mathbf{e}_\theta \]
\[ = \left[ \frac{\pi^2}{2} - (\pi)^2 \right] \mathbf{e}_r + \left[ 2 \left( -\frac{\pi}{2} \right) (\pi) \right] \mathbf{e}_\theta \]

or
\[ \mathbf{a} = -\left( \frac{\pi^2}{2} \text{ m/s}^2 \right) \mathbf{e}_r - (\pi^2 \text{ m/s}^2) \mathbf{e}_\phi \]

\[ \text{ } \]
PROBLEM 11.166

The two-dimensional motion of a particle is defined by the relations \( r = 2a \cos \theta \) and \( \theta = \frac{1}{2} b t^2 \), where \( a \) and \( b \) are constants. Determine (a) the magnitudes of the velocity and acceleration at any instant, (b) the radius of curvature of the path. What conclusion can you draw regarding the path of the particle?

SOLUTION

(a) We have

\[
\begin{align*}
\mathbf{r} &= 2a \cos \theta \\
\mathbf{\theta} &= \frac{1}{2} b t^2
\end{align*}
\]

Then

\[
\begin{align*}
\mathbf{r}' &= -2a \theta \sin \theta \\
\mathbf{\dot{\theta}} &= b t
\end{align*}
\]

and

\[
\begin{align*}
\mathbf{\ddot{r}} &= -2a (\dot{\theta} \sin \theta + \dot{\theta}^2 + \cos \theta) \\
\mathbf{\ddot{\theta}} &= b
\end{align*}
\]

Substituting for \( \dot{\theta} \) and \( \ddot{\theta} \)

\[
\begin{align*}
\mathbf{r} &= -2abt \sin \theta \\
\mathbf{\ddot{r}} &= -2ab(\sin \theta + bt^2 \cos \theta)
\end{align*}
\]

Now

\[
\begin{align*}
\mathbf{v}_r &= r = -2abt \sin \theta \\
\mathbf{v}_\theta &= \mathbf{\dot{\theta}} = 2abt \cos \theta
\end{align*}
\]

Then

\[
\mathbf{v} = \sqrt{\mathbf{v}_r^2 + \mathbf{v}_\theta^2} = 2abt [(-\sin \theta)^2 + (\cos \theta)^2]^{1/2}
\]

or

\[
\mathbf{v} = 2abt \quad \nabla
\]

Also

\[
\mathbf{a}_r = r - \dot{\mathbf{r}}^2 = -2ab(\sin \theta + bt^2 \cos \theta) - 2ab^2t^2 \cos \theta
\]

\[
= -2ab(\sin \theta + 2bt^2 \cos \theta)
\]

and

\[
\mathbf{a}_\theta = \dot{\mathbf{r}} + 2r \dot{\mathbf{\theta}} = 2ab \cos \theta - 4ab^2t^2 \sin \theta
\]

\[
= -2ab(\cos \theta - 2bt^2 \sin \theta)
\]

Then

\[
\mathbf{a} = \sqrt{\mathbf{a}_r^2 + \mathbf{a}_\theta^2} = 2ab [(\sin \theta + 2bt^2 \cos \theta)^2
\]

\[
+ (\cos \theta - 2bt^2 \sin \theta)^2]^{1/2}
\]

or

\[
\mathbf{a} = 2ab \sqrt{1 + 4b^2t^4} \quad \nabla
\]

(b) Now

\[
\mathbf{a}^2 = \mathbf{a}_r^2 + \mathbf{a}_\theta^2 = \left( \frac{\mathbf{d}v}{\mathbf{d}t} \right)^2 + \left( \frac{\mathbf{v}^2}{\rho} \right)^2
\]

Then

\[
\frac{\mathbf{d}v}{\mathbf{d}t} = \frac{\mathbf{d}}{\mathbf{d}t} (2abt) = 2ab
\]
PROBLEM 11.166 (Continued)

so that
\[
\left(2ab\sqrt{1 + 4b^2t^4}\right)^2 = (2ab)^2 + a_n^2
\]
or
\[
4a^2b^2(1 + 4b^2t^4) = 4a^2b^2 + a_n^2
\]
or
\[
a_n = 4ab^2t^2
\]
Finally
\[
a_n = \frac{\sqrt{2}}{\rho} \Rightarrow \rho = \frac{(2abt)^2}{4ab^2t^2}
\]
or
\[
\rho = a
\]

Since the radius of curvature is a constant, the path is a circle of radius a.
**PROBLEM 11.167**

To study the performance of a race car, a high-speed motion-picture camera is positioned at Point A. The camera is mounted on a mechanism which permits it to record the motion of the car as the car travels on straightway BC. Determine the speed of the car in terms of \( b \), \( \theta \), and \( \dot{\theta} \).

---

**SOLUTION**

We have

\[
\mathbf{r} = \frac{b}{\cos \theta}
\]

Then

\[
\mathbf{r} = \frac{b \dot{\theta} \sin \theta}{\cos^2 \theta}
\]

We have

\[
\mathbf{v}^2 = \mathbf{v}_r^2 + \mathbf{v}_q^2 = (r)^2 + (r \dot{\theta})^2 = \left( \frac{b \dot{\theta} \sin \theta}{\cos^2 \theta} \right)^2 + \left( \frac{b \dot{\theta} \sin \theta}{\cos \theta} \right)^2 = \frac{b^2 \dot{\theta}^2 \sin^2 \theta}{\cos^2 \theta} + \frac{b^2 \dot{\theta}^2 \sin^2 \theta}{\cos \theta} = \frac{b^2 \dot{\theta}^2}{\cos^2 \theta} + 1
\]

or

\[
\mathbf{v} = \pm \frac{b \dot{\theta}}{\cos \theta}
\]

For the position of the car shown, \( \theta \) is decreasing; thus, the negative root is chosen.

\[
\mathbf{v} = \frac{b \dot{\theta}}{\cos \theta}
\]

Alternative solution.

From the diagram

\[
\mathbf{r} = -v \sin \theta
\]

or

\[
\frac{b \dot{\theta} \sin \theta}{\cos^2 \theta} = -v \sin \theta
\]

or

\[
\mathbf{v} = \frac{b \dot{\theta}}{\cos \theta}
\]
**PROBLEM 11.168**

Determine the magnitude of the acceleration of the race car of Problem 11.167 in terms of \( b \), \( \theta \), \( \dot{\theta} \), and \( \ddot{\theta} \).

---

**SOLUTION**

We have

\[
r = \frac{b}{\cos \theta}
\]

Then

\[
r' = \frac{b\dot{\theta} \sin \theta}{\cos^2 \theta}
\]

For rectilinear motion

\[
a = \frac{dv}{dt}
\]

From the solution to Problem 11.167

\[
v = -\frac{b\dot{\theta}}{\cos \theta}
\]

Then

\[
a = \frac{d}{dt}\left(-\frac{b\dot{\theta}}{\cos \theta}\right) = -\frac{b\dot{\theta} \cos^2 \theta - \dot{\theta}(-2\dot{\theta} \cos \theta \sin \theta)}{\cos^4 \theta}
\]

or

\[
a = -\frac{b}{\cos^2 \theta}(\ddot{\theta} + 2\dot{\theta}^2 \tan \theta)
\]

Alternative solution

From above

\[
r = \frac{b}{\cos \theta}
\]

Then

\[
r = b\left(\frac{\dot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta - (\dot{\theta} \sin \theta)(-2\dot{\theta} \cos \theta \sin \theta)}{\cos^4 \theta}\right)
\]

\[
= b\left[\frac{\dot{\theta} \sin \theta}{\cos^2 \theta} + \frac{\dot{\theta}^2 (1 + \sin^2 \theta)}{\cos^3 \theta}\right]
\]

---

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PROBLEM 11.168 (Continued)

Now

\[ a^2 = a_x^2 + a_y^2 \]

where

\[ a_x = r - r\dot{\theta}^2 = b \left[ \dot{\theta} \frac{\sin \theta}{\cos^2 \theta} + \frac{\dot{\theta}^2(1 + \sin^2 \theta)}{\cos^2 \theta} \right] - \frac{b\dot{\theta}^2}{\cos \theta} \]

\[ = \frac{b}{\cos^2 \theta} \left( \frac{\dot{\theta} \sin \theta + 2\dot{\theta}^2 \sin^2 \theta}{\cos \theta} \right) \]

\[ a_x = \frac{b \sin \theta}{\cos^2 \theta} (\dot{\theta} + 2\dot{\theta}^2 \tan \theta) \]

and

\[ a_y = r\ddot{\theta} + 2r\dot{\theta} = \frac{b\ddot{\theta}}{\cos \theta} + 2 \frac{b\dot{\theta}^2 \sin \theta}{\cos^2 \theta} \]

\[ = \frac{b \cos \theta}{\cos^2 \theta} (\ddot{\theta} + 2\dot{\theta}^2 \tan \theta) \]

Then

\[ a = \pm \frac{b}{\cos^2 \theta} (\dot{\theta} + 2\dot{\theta}^2 \tan \theta) \left[ (\sin \theta)^2 + (\cos \theta)^2 \right]^{1/2} \]

For the position of the car shown, \( \ddot{\theta} \) is negative; for \( a \) to be positive, the negative root is chosen.

\[ a = -\frac{b}{\cos^2 \theta} (\dot{\theta} + 2\dot{\theta}^2 \tan \theta) \]
PROBLEM 11.169

After taking off, a helicopter climbs in a straight line at a constant angle $\beta$. Its flight is tracked by radar from Point A. Determine the speed of the helicopter in terms of $d$, $\beta$, $\theta$, and $\dot{\theta}$.

SOLUTION

From the diagram

$$\frac{r}{\sin(180^\circ - \beta)} = \frac{d}{\sin(\beta - \theta)}$$

or

$$d \sin \beta = r(\sin \beta \cos \theta - \cos \beta \sin \theta)$$

or

$$r = d \frac{\tan \beta}{\tan \beta \cos \theta - \sin \theta}$$

Then

$$r = d \tan \beta \frac{(-\tan \beta \sin \theta - \cos \theta)}{(\tan \beta \cos \theta - \sin \theta)^2} \dot{\theta}$$

$$= d \theta \tan \beta \frac{\tan \beta \sin \theta + \cos \theta}{(\tan \beta \cos \theta - \sin \theta)^2}$$

From the diagram

$$v_r = v \cos (\beta - \theta)$$

where

$$v_r = r$$

Then

$$d \dot{\theta} \tan \beta \frac{\tan \beta \sin \theta + \cos \theta}{(\tan \beta \cos \theta - \sin \theta)^2} = v(\cos \beta \cos \theta + \sin \beta \sin \theta)$$

$$= v \cos \beta (\tan \beta \sin \theta + \cos \theta)$$

or

$$v = \frac{d \dot{\theta} \tan \beta \sec \beta}{(\tan \beta \cos \theta - \sin \theta)^2}$$

Alternative solution.

We have

$$v^2 = v_r^2 + v_\theta^2 = (r)^2 + (r \dot{\theta})^2$$
PROBLEM 11.169 (Continued)

Using the expressions for \( r \) and \( \dot{r} \) from above

\[
v = \left[ \frac{d\dot{\theta}}{\tan \beta} \frac{\tan \beta \sin \theta + \cos \theta}{(\tan \beta \cos \theta - \sin \theta)^2} \right]^2
\]

or

\[
v = \pm \frac{d\dot{\theta} \tan \beta}{(\tan \beta \cos \theta - \sin \theta)} \left[ \frac{(\tan \beta \sin \theta + \cos \theta)^2}{(\tan \beta \cos \theta - \sin \theta)^2} + 1 \right]^{1/2}
\]

\[
= \pm \frac{d\dot{\theta} \tan \beta}{(\tan \beta \cos \theta - \sin \theta)} \left[ \frac{\tan^2 \beta + 1}{(\tan \beta \cos \theta - \sin \theta)^2} \right]^{1/2}
\]

Note that as \( \theta \) increases, the helicopter moves in the indicated direction. Thus, the positive root is chosen.

\[
v = \frac{d\dot{\theta} \tan \beta \sec \beta}{(\tan \beta \cos \theta - \sin \theta)^2}
\]
SOLUTION

From the diagram

\[
\frac{r}{\sin(90^\circ - \beta)} = \frac{h}{\sin(\beta + \theta)}
\]

or

\[r(\sin \beta \cos \theta + \cos \beta \sin \theta) = h \cos \beta\]

or

\[r = \frac{h}{\tan \beta \cos \theta + \sin \theta}\]

Also,

\[v_0 = v_0 \sin(\beta + \theta) \quad \text{where} \quad v_0 = \dot{r}\]

Then

\[\frac{\dot{h}}{\tan \beta \cos \theta + \sin \theta} = v_0(\sin \beta \cos \theta + \cos \beta \sin \theta)\]

or

\[\dot{\theta} = \frac{v_0 \cos \beta}{h} (\tan \beta \cos \theta + \sin \theta)^2\]

Alternative solution.

From above

\[r = \frac{h}{\tan \beta \cos \theta + \sin \theta}\]

Then

\[r = h \frac{\tan \beta \cos \theta - \sin \theta}{(\tan \beta \cos \theta + \sin \theta)^2} \dot{\theta}\]

Now

\[v_0^2 = v_x^2 + v_y^2 = (r)^2 + (r \dot{\theta})^2\]

or

\[v_0^2 = \left[\frac{\dot{h}}{\tan \beta \sin \theta - \cos \theta}{(\tan \beta \cos \theta + \sin \theta)^2}\right]^2 + \left(\frac{\dot{h}}{\tan \beta \cos \beta + \sin \theta}\right)^2\]
PROBLEM 11.170 (Continued)

or

\[ v_0 = \pm \frac{h \dot{\theta}}{\tan \beta \cos \theta + \sin \theta} \]

\[ + \left[ \frac{(\tan \beta \sin \theta - \cos \theta)^2}{(\tan \beta \cos \theta + \sin \theta)^2 + 1} \right]^{1/2} \]

\[ = \pm \frac{h \dot{\theta}}{\tan \beta \cos \theta + \sin \theta} \left[ \frac{\tan^2 \beta + 1}{(\tan \beta \cos \theta + \sin \theta)^2} \right]^{1/2} \]

Note that as \( \theta \) increases, member BC moves in the indicated direction. Thus, the positive root is chosen.

\[ \dot{\theta} = \frac{v_0 \cos \beta}{h} (\tan \beta \cos \theta + \sin \theta)^2 \]

\[ \Rightarrow \]
**PROBLEM 11.171**

For the race car of Problem 11.167, it was found that it took 0.5 s for the car to travel from the position $\theta = 60^\circ$ to the position $\theta = 35^\circ$. Knowing that $b = 25$ m, determine the average speed of the car during the 0.5-s interval.

**SOLUTION**

From the diagram:

\[
\Delta \theta_2 = 25 \tan 60^\circ - 25 \tan 35^\circ = 25.796 \text{ m}
\]

Now

\[
\nu_{ave} = \frac{\Delta \theta_2}{\Delta t_2} = \frac{25.796 \text{ m}}{0.5 \text{ s}} = 51.592 \text{ m/s}
\]

or

\[
\nu_{ave} = 185.7 \text{ km/h}
\]
PROBLEM 11.172

For the helicopter of Problem 11.169, it was found that when the helicopter was at B, the distance and the angle of elevation of the helicopter were \( r = 1000 \text{ m} \) and \( \theta = 20^\circ \), respectively. Four seconds later, the radar station sighted the helicopter at \( r = 1100 \text{ m} \) and \( \theta = 23.1^\circ \). Determine the average speed and the angle of climb \( \beta \) of the helicopter during the 4-s interval.

SOLUTION

We have
\[
\begin{align*}
  r_0 &= 1000 \text{ m} \quad \theta_0 = 20^\circ \\
  r_4 &= 1100 \text{ m} \quad \theta_4 = 23.1^\circ 
\end{align*}
\]

From the diagram:
\[
\begin{align*}
  \Delta r^2 &= 1000^2 + 1100^2 \\
  &\quad - 2(1000)(1100) \cos (23.1^\circ - 20^\circ) \\
  \text{or} \\
  \Delta r &= 114.975 \text{ m}
\end{align*}
\]

Now
\[
\begin{align*}
  v_{\text{ave}} &= \frac{\Delta r}{\Delta t} \\
  &= \frac{114.975 \text{ m}}{4 \text{ s}} \\
  &= 28.74375 \text{ m/s}
\end{align*}
\]

or
\[
  v_{\text{ave}} = 103.5 \text{ km/h} \quad \blacktriangleleft
grey
\]

Also,
\[
\begin{align*}
  \Delta r \cos \beta &= r_4 \cos \theta_4 - r_0 \cos \theta_0 \\
  \text{or} \\
  \cos \beta &= \frac{1100 \cos 23.1^\circ - 1000 \cos 20^\circ}{114.975}
\end{align*}
\]

or
\[
  \beta = 51.2^\circ \quad \blacktriangleleft
\]
PROBLEM 11.173

A particle moves along the spiral shown; determine the magnitude of the velocity of the particle in terms of $b$, $\theta$, and $\dot{\theta}$.

SOLUTION

Hyperbolic spiral.

$$r = \frac{b}{\theta}$$

$$\frac{dr}{dt} = -\frac{b}{\theta^2}$$

$$v_r = r \frac{dr}{dt} = -\frac{b}{\theta^2}$$

$$v_\theta = \frac{b}{\theta}$$

$$v = \sqrt{v_r^2 + v_\theta^2} = \frac{b\theta}{\theta^2} \sqrt{\left(\frac{1}{\theta^2}\right)^2 + \left(\frac{1}{\theta}\right)^2}$$

$$v = \frac{b\theta}{\theta^2} \sqrt{1 + \theta^2}$$

$$v = \frac{b}{\theta^2} \sqrt{1 + \theta^2} \dot{\theta}$$
PROBLEM 11.174

A particle moves along the spiral shown; determine the magnitude of the velocity of the particle in terms of \( b \), \( \theta \), and \( \dot{\theta} \).

SOLUTION

Logarithmic spiral. 

\[
\dot{r} = \frac{dr}{dt} = be^{b\theta} \frac{d\theta}{dt} = be^{b\theta} \dot{\theta}
\]

\[
\dot{v}_r = r = be^{b\theta} \quad \dot{v}_\theta = r \dot{\theta} = e^{b\theta} \dot{\theta}
\]

\[
v = \sqrt{\dot{v}_r^2 + \dot{v}_\theta^2} = e^{b\theta} \dot{\theta} \sqrt{b^2 + 1}
\]

\[v = e^{b\theta} \sqrt{1 + b^2 \dot{\theta}^2}\]
**PROBLEM 11.175**

A particle moves along the spiral shown. Knowing that \( \dot{\theta} \) is constant and denoting this constant by \( \omega \), determine the magnitude of the acceleration of the particle in terms of \( b \), \( \theta \), and \( \omega \).

---

**SOLUTION**

Hyperbolic spiral.

\[ r = \frac{b}{\theta} \]

From Problem 11.173

\[ r = -\frac{b}{\theta^2} \ddot{\theta} \]

\[ r = -\frac{b}{\theta^2} \ddot{\theta} + \frac{2b}{\theta^3} \dot{\theta}^2 \]

\[ a_r = r - r\ddot{\theta}^2 = -\frac{b}{\theta^2} \ddot{\theta} + \frac{2b}{\theta^3} \dot{\theta}^2 - \frac{b}{\theta^2} \dot{\theta}^2 \]

\[ a_r = r\ddot{\theta} + 2r\dot{\theta} = \frac{b}{\theta} \dddot{\theta} + 2\left( -\frac{b}{\theta^2} \ddot{\theta} \right) \dot{\theta} = \frac{b}{\theta} \dddot{\theta} - 2\frac{b}{\theta^2} \dot{\theta}^2 \]

Since \( \dot{\theta} = \omega = \text{constant} \) and \( \ddot{\theta} = 0 \), and we write:

\[ a_r = \frac{2b}{\theta^3} \omega^2 - \frac{b}{\theta} \omega^2 = \frac{b\omega^2}{\theta^3} (2 - \theta^2) \]

\[ a_r = -2\frac{b}{\theta^2} \omega^2 = -\frac{b\omega^2}{\theta^3} (2\theta) \]

\[ a = \sqrt{a_r^2 + a_r^2} = \frac{b\omega^2}{\theta^3} \sqrt{(2 - \theta^2)^2 + (2\theta)^2} = \frac{b\omega^2}{\theta^3} \sqrt{4 - 4\theta^2 + \theta^4 + 4\theta^4} \]

\[ a = \frac{b\omega^2}{\theta^3} \sqrt{4 + \theta^4} \]
**PROBLEM 11.176**

A particle moves along the spiral shown. Knowing that \( \dot{\theta} \) is constant and denoting this constant by \( \omega \), determine the magnitude of the acceleration of the particle in terms of \( b \), \( \theta \), and \( \omega \).

**SOLUTION**

Logarithmic spiral,

\[
r = e^{b \theta}
\]

\[
\dot{r} = \frac{dr}{dt} = be^{b \theta} \dot{\theta}
\]

\[
\ddot{r} = \dot{r} - r \ddot{\theta} = be^{b \theta} \dot{\theta} + b^2 e^{b \theta} \ddot{\theta} = be^{b \theta} (\dot{\theta} + b \ddot{\theta})
\]

\[
a_r = r - r \ddot{\theta} = be^{b \theta} (\dot{\theta} + b \ddot{\theta}) - e^{b \theta} \ddot{\theta}^2
\]

\[
a_\theta = r \ddot{\theta} + 2r \dot{\theta} = e^{b \theta} \ddot{\theta} + 2(be^{b \theta} \dot{\theta}) \dot{\theta}
\]

Since \( \dot{\theta} = \omega = \text{constant} \), \( \ddot{\theta} = \theta \), and we write

\[
a_r = be^{b \theta} (b \omega^2) - e^{b \theta} \omega^2 = e^{b \theta} (b^2 - 1) \omega^2
\]

\[
a_\theta = 2be^{b \theta} \omega^2
\]

\[
a = \sqrt{a_r^2 + a_\theta^2} = e^{b \theta} \omega^2 \sqrt{(b^2 - 1)^2 + (2b)^2}
\]

\[
= e^{b \theta} \omega^2 \sqrt{b^4 - 2b^2 + 1 + 4b^2} = e^{b \theta} \omega^2 \sqrt{b^4 + 2b^2 + 1}
\]

\[
= e^{b \theta} \omega^2 \sqrt{(b^2 + 1)^2} = e^{b \theta} \omega^2 (b^2 + 1)
\]

\[
a = (1 + b^2) \omega^2 e^{b \theta}
\]
PROBLEM 11.177

Show that \( \dot{r} = h \dot{\theta} \sin \theta \) knowing that at the instant shown, step AB of the step exerciser is rotating counterclockwise at a constant rate \( \dot{\theta} \).

SOLUTION

From the diagram

\[
\begin{align*}
    r^2 &= d^2 + h^2 - 2dh \cos \phi \\
    \text{Then} & \\
    2rr &= 2dh \dot{\phi} \sin \phi \\
    \text{Now} & \\
    \frac{r}{\sin \phi} &= \frac{d}{\sin \theta} \\
    \text{or} & \\
    r &= \frac{ds \sin \phi}{\sin \theta} \\
\end{align*}
\]

Substituting for \( r \) in the expression for \( \dot{r} \)

\[
\begin{align*}
    \left( \frac{ds \sin \phi}{\sin \theta} \right) r &= dh \dot{\phi} \sin \phi \\
    \text{or} & \\
    \dot{r} &= h \dot{\phi} \sin \theta \\
\end{align*}
\]

Q.E.D.

Alternative solution.

First note

\[
\alpha = 180^\circ - (\phi + \theta)
\]

Now

\[
\mathbf{v} = \mathbf{v}_r + \mathbf{v}_\theta = r \dot{\mathbf{e}}_r + r \dot{\phi} \mathbf{e}_\phi
\]

With B as the origin

\[
\mathbf{v}_p = d \dot{\phi} \quad (d = \text{constant} \Rightarrow \dot{d} = 0)
\]
PROBLEM 11.177 (Continued)

With O as the origin

\( (v_P)_r = r \)

where

\( (v_P)_r = v_P \sin \alpha \)

Then

\( r = d\phi \sin \alpha \)

Now

\( \frac{h}{\sin \alpha} = \frac{d}{\sin \theta} \)

or

\( d\sin \alpha = h\sin \theta \)
**SOLUTION**

We have

\[ R = A, \quad \theta = 2\pi t, \quad z = \frac{1}{4} At^2 \]

Then

\[ R = 0, \quad \dot{\theta} = 2\pi, \quad z = \frac{1}{2} At \]

and

\[ R = 0, \quad \ddot{\theta} = 0, \quad z = \frac{1}{2} A \]

Now

\[ v^2 = v_R^2 + v_\theta^2 + v_z^2 \]

\[ = (R\dot{\theta})^2 + (R\ddot{\theta})^2 + (z)^2 \]

\[ = 0 + (A + 2\pi)^2 + \left(\frac{1}{2} At\right)^2 \]

\[ = A^2 \left(4\pi^2 + \frac{1}{4} t^2\right) \]

or

\[ v = \frac{1}{2} A\sqrt{16\pi^2 + t^2} \]

and

\[ a^2 = a_R^2 + a_\theta^2 + a_z^2 \]

\[ = (R\ddot{\theta} + R\dot{\theta}^2)^2 + (R\dddot{\theta} + 2R\ddot{\theta})^2 + (z)^2 \]

\[ = [-A(2\pi t)^2 + 0 + \left(\frac{1}{2} A\right)^2] \]

\[ = A^2 \left(16\pi^4 + \frac{1}{4}\right) \]

or

\[ a = \frac{1}{2} A\sqrt{64\pi^4 + 1} \]
**PROBLEM 11.179**

The three-dimensional motion of a particle is defined by the cylindrical coordinates (see Figure 11.26) $R = A/(t+1)$, $\theta = Bt$, and $z = Ct/(t+1)$. Determine the magnitudes of the velocity and acceleration when (a) $t = 0$, (b) $t = \infty$.

**SOLUTION**

We have

\[
R = \frac{A}{t+1}, \quad \theta = Bt, \quad z = \frac{Ct}{t+1}
\]

Then

\[
\dot{R} = -\frac{A}{(t+1)^2}, \quad \dot{\theta} = B, \quad \dot{z} = \frac{C(t+1)-t}{(t+1)^2}
\]

and

\[
\ddot{R} = -\frac{2A}{(t+1)^3}, \quad \ddot{\theta} = 0, \quad \ddot{z} = -\frac{2C}{(t+1)^3}
\]

Now

\[
v^2 = (v_R)^2 + (v_\theta)^2 + (v_z)^2
\]

\[
= (R)^2 + (R\dot{\theta})^2 + (z)^2
\]

and

\[
a^2 = (a_R)^2 + (a_\theta)^2 + (a_z)^2
\]

\[
= (R - R\ddot{\theta})^2 + (R\ddot{\theta}^0 + 2R\dot{\theta})^2 + (z)^2
\]

(a) At $t = 0$:

\[
R = A
\]

\[
R = -A \quad \theta = B \quad z = C
\]

\[
R = 2A \quad z = -2C
\]

Then

\[
v^2 = (-A)^2 + (AB)^2 + (C)^2
\]

or

\[
v = \sqrt{(1 + B^2)A^2 + C^2}
\]

and

\[
a^2 = (2A - AB^2)^2 + (z(-A)(B))^2 + (-2C)^2
\]

\[
= 4A^2 \left[ \left(1 - \frac{1}{2}B^2 \right)^2 + B^2 + \frac{C^2}{A^2} \right]
\]

\[
= 4 \left[ \left(1 + \frac{1}{4}B^4 \right)A^2 + C^2 \right]
\]

or

\[
a = 2\sqrt{\left(1 + \frac{1}{4}B^4 \right)A^2 + C^2}
\]
**PROBLEM 11.179 (Continued)**

(b) As \( t \to \infty \):

\[
R = 0 \\
\dot{R} = 0 \quad \dot{\theta} = B \quad z = 0 \\
\ddot{R} = 0 \quad \ddot{z} = 0
\]

\[v = 0\]
\[a = 0\]
PROBLEM 11.180*

For the conic helix of Problem 11.95, determine the angle that the osculating plane forms with the y axis.

SOLUTION

First note that the vectors \( \mathbf{v} \) and \( \mathbf{a} \) lie in the osculating plane.

Now

\[
\mathbf{r} = (R \cos \omega_n t) \mathbf{i} + c \mathbf{j} + (R \sin \omega_n t) \mathbf{k}
\]

Then

\[
\mathbf{v} = \frac{d\mathbf{r}}{dt} = R(\cos \omega_n t - \omega_n t \sin \omega_n t) \mathbf{i} + c + R(\sin \omega_n t + \omega_n t \cos \omega_n t) \mathbf{k}
\]

and

\[
\mathbf{a} = \frac{d\mathbf{v}}{dt} = -R(\omega_n \sin \omega_n t - \omega_n \sin \omega_n t - \omega_n^2 t \cos \omega_n t) \mathbf{i} + R(\omega_n \cos \omega_n t + \omega_n \cos \omega_n t - \omega_n^2 t \sin \omega_n t) \mathbf{k}
\]

\[
= \omega_n R[-(2 \sin \omega_n t + \omega_n t \cos \omega_n t) \mathbf{i} + (2 \cos \omega_n t - \omega_n t \sin \omega_n t) \mathbf{k}]
\]

It then follows that the vector \( (\mathbf{v} \times \mathbf{a}) \) is perpendicular to the osculating plane.

\[
(\mathbf{v} \times \mathbf{a}) = \omega_n R \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ R(\cos \omega_n t - \omega_n t \sin \omega_n t) & c & R(\sin \omega_n t + \omega_n t \cos \omega_n t) \\ -(2 \sin \omega_n t + \omega_n t \cos \omega_n t) & 0 & (2 \cos \omega_n t - \omega_n t \sin \omega_n t) \end{vmatrix}
\]

\[
= \omega_n R[c(2 \cos \omega_n t - \omega_n t \sin \omega_n t) \mathbf{i} + R[-(\sin \omega_n t + \omega_n t \cos \omega_n t)(2 \sin \omega_n t + \omega_n t \cos \omega_n t) \\
- (\cos \omega_n t - \omega_n t \sin \omega_n t)(2 \cos \omega_n t - \omega_n t \sin \omega_n t)] \mathbf{j} + c(2 \sin \omega_n t + \omega_n t \cos \omega_n t) \mathbf{k}
\]

\[
= \omega_n R[c(2 \cos \omega_n t - \omega_n t \sin \omega_n t) \mathbf{i} - R(2 + \omega_n^2 t^2) \mathbf{j} + c(2 \sin \omega_n t + \omega_n t \cos \omega_n t) \mathbf{k}]
\]
**PROBLEM 11.180* (Continued)**

The angle \( \alpha \) formed by the vector \((v \times a)\) and the y axis is found from

\[
\cos \alpha = \frac{(v \times a) \cdot j}{|(v \times a)| \cdot |j|}
\]

where

\[
|(v \times a)| = \omega_n R^2 \left(2 + \omega_n^2 t^2\right)
\]

\[
(v \times a) \cdot j = -\omega_n R^2 \left(2 + \omega_n^2 t^2\right)
\]

Then

\[
\cos \alpha = \frac{-\omega_n R^2 \left(2 + \omega_n^2 t^2\right)}{\omega_n R^2 \left[ c^2 (4 + \omega_n^2 t^2) + R^2 \left(2 + \omega_n^2 t^2\right)^2 \right]^{1/2}}
\]

The angle \( \beta \) that the osculating plane forms with y axis (see the above diagram) is equal to

\[
\beta = \alpha - 90^\circ
\]

Then

\[
\cos \alpha = \cos (\beta + 90^\circ) = -\sin \beta
\]

\[
-\sin \beta = \frac{-R \left(2 + \omega_n^2 t^2\right)}{\left[ c^2 (4 + \omega_n^2 t^2) + R^2 \left(2 + \omega_n^2 t^2\right)^2 \right]^{1/2}}
\]

Then

\[
\tan \beta = \frac{R \left(2 + \omega_n^2 t^2\right)}{c \sqrt{4 + \omega_n^2 t^2}}
\]

or

\[
\beta = \tan^{-1} \left[ \frac{R \left(2 + \omega_n^2 t^2\right)}{c \sqrt{4 + \omega_n^2 t^2}} \right]
\]
**PROBLEM 11.181***

Determine the direction of the binormal of the path described by the particle of Problem 11.96 when (a) \( t = 0 \), (b) \( t = \pi/2 \) s.

---

**SOLUTION**

Given:

\[ \mathbf{r} = (At \cos t)\mathbf{i} + \left(A\sqrt{t^2 + 1}\right)\mathbf{j} + (Bt \sin t)\mathbf{k} \]

\( r - m, \ t - s; \ A = 3, \ B = 1 \)

First note that \( \mathbf{e}_b \) is given by

\[ \mathbf{e}_b = \frac{\mathbf{v} \times \mathbf{a}}{|\mathbf{v} \times \mathbf{a}|} \]

Now

\[ \mathbf{r} = (3 \cos t)\mathbf{i} + \left(3\sqrt{t^2 + 1}\right)\mathbf{j} + (t \sin t)\mathbf{k} \]

Then

\[ \mathbf{v} = \frac{d\mathbf{r}}{dt} \]

\[ = 3(\cos - t \sin t)\mathbf{i} + \frac{3}{\sqrt{t^2 + 1}} \mathbf{j} + (\sin + t \cos t)\mathbf{k} \]

and

\[ \mathbf{a} = \frac{d\mathbf{v}}{dt} \]

\[ = 3(- \sin - \sin t - t \cos t)\mathbf{i} + 3\sqrt{t^2 + 1} - t \left(\frac{t}{\sqrt{t^2 + 1}}\right) \mathbf{j} + (\cos + \cos t - t \sin t)\mathbf{k} \]

\[ = -3(2 \sin t + t \cos t)\mathbf{i} + \frac{3}{(t^2 + 1)^{3/2}} \mathbf{j} + (2 \cos t - t \sin t)\mathbf{k} \]

(a) \( \text{At } t = 0: \)

\[ \mathbf{v} = (3 \text{ m/s})\mathbf{i} \]

\[ \mathbf{a} = (3 \text{ m/s}^2)\mathbf{j} + (2 \text{ m/s}^2)\mathbf{k} \]

Then

\[ \mathbf{v} \times \mathbf{a} = 3 \times (3 \mathbf{j} + 2 \mathbf{k}) \]

\[ = 3(-2 \mathbf{j} + 3 \mathbf{k}) \]

and

\[ |\mathbf{v} \times \mathbf{a}| = 3\sqrt{(-2)^2 + (3)^2} = 3/13 \]

Then

\[ \mathbf{e}_b = \frac{3(-2 \mathbf{j} + 3 \mathbf{k})}{3/13} = \frac{1}{\sqrt{13}}(-2 \mathbf{j} + 3 \mathbf{k}) \]

\[ \cos \theta_x = 0 \quad \cos \theta_y = -\frac{2}{\sqrt{13}} \quad \cos \theta_z = \frac{3}{\sqrt{13}} \]

or

\( \theta_x = 90^o \quad \theta_y = 123.7^o \quad \theta_z = 33.7^o \)
PROBLEM 11.181* (Continued)

(b) At $t = \frac{\pi}{2}$

$v = -\left(\frac{3\pi}{2} \text{ m/s}\right)i + \left(\frac{3\pi}{\sqrt{\pi^2 + 4}} \text{ m/s}\right)j + (1 \text{ m/s})k$

$a = -(6 \text{ m/s}^2)i + \left[\frac{24}{(\pi^2 + 4)^{3/2}} \text{ m/s}^2\right]j - \left(\frac{\pi}{2} \text{ m/s}^2\right)k$

Then

$v \times a = \begin{vmatrix} i & j & k \\ \frac{3\pi}{2} & \frac{3\pi}{(\pi^2 + 4)^{3/2}} & 1 \\ -6 & \frac{24}{(\pi^2 + 4)^{3/2}} & -\frac{\pi}{2} \end{vmatrix} = -\left[\frac{3\pi^2}{2(\pi^2 + 4)^{3/2}} + \frac{24}{(\pi^2 + 4)^{3/2}}\right]i - \left(6 + \frac{3\pi^2}{4}\right)j

+ \left[-\frac{36\pi}{(\pi^2 + 4)^{3/2}} + \frac{18\pi}{(\pi^2 + 4)^{3/2}}\right]k

= -4.43984i - 13.40220 + 12.99459k$

and $|v \times a| = \sqrt{(-4.43984)^2 + (-13.40220)^2 + (12.99459)^2} = 19.18829$

Then

$e_b = \frac{1}{19.1829}(-4.43984i - 13.40220j + 12.99459k)$

$\cos \theta_x = \frac{4.43984}{19.1829} \quad \cos \theta_y = \frac{-13.40220}{19.1829} \quad \cos \theta_z = \frac{12.99459}{19.1829}$

or $\theta_x = 103.4^\circ \quad \theta_y = 134.3^\circ \quad \theta_z = 47.4^\circ$
**PROBLEM 11.182**

The motion of a particle is defined by the relation \( x = 2t^3 - 15t^2 + 24t + 4 \), where \( x \) and \( t \) are expressed in meters and seconds, respectively. Determine (a) when the velocity is zero, (b) the position and the total distance traveled when the acceleration is zero.

---

**SOLUTION**

\[
x = 2t^3 - 15t^2 + 24t + 4
\]

so

\[
v = \frac{dx}{dt} = 6t^2 - 30t + 24
\]

\[
a = \frac{dv}{dt} = 12t - 30
\]

(a) **Times when \( v = 0 \).**

\[
0 = 6t^2 - 30t + 24 = 6(t^2 - 5t + 4)
\]

\[
(t - 4)(t - 1) = 0 \quad t = 1.00 \text{ s}, \quad t = 4.00 \text{ s}
\]

(b) **Position and distance traveled when \( a = 0 \).**

\[
a = 12t - 30 = 0 \quad t = 2.5 \text{ s}
\]

so

\[
x_2 = 2(2.5)^3 - 15(2.5)^2 + 24(2.5) + 4
\]

**Final position**

\[
x = 1.50 \text{ m}
\]

- For \( 0 < t < 1 \text{ s}, \quad v > 0 \).
- For \( 1 \text{ s} < t < 2.5 \text{ s}, \quad v < 0 \).
- At \( t = 0, \quad x_0 = 4 \text{ m} \).
- At \( t = 1 \text{ s}, \quad x_1 = (2)(1)^3 - (15)(1)^2 + (24)(1) + 4 = 15 \text{ m} \)

**Distance traveled over interval:** \( x_1 - x_0 = 11 \text{ m} \)

- For \( 1 \text{ s} < t < 2.5 \text{ s}, \quad v < 0 \)

**Distance traveled over interval**

\[ |x_2 - x_1| = |1.5 - 15| = 13.5 \text{ m} \]

**Total distance:**

\[
d = 11 + 13.5 \quad d = 24.5 \text{ m}
\]
**PROBLEM 11.183**

The acceleration of a particle is defined by the relation \( a = -60x^{1.5} \), where \( a \) and \( x \) are expressed in m/s\(^2\) and meters, respectively. Knowing that the particle starts with no initial velocity at \( x = 4 \) m, determine the velocity of the particle when (a) \( x = 2 \) m, (b) \( x = 1 \) m, (c) \( x = 100 \) mm.

**SOLUTION**

We have \( \frac{dv}{dx} = a = -60x^{1.5} \).

When \( x = 4 \) m, \( v = 0 \):

\[
\int_0^v dv = \int_4^x (-60x^{1.5}) \, dx
\]

or

\[
\frac{1}{2}v^2 = 120(x^{0.5})^2
\]

or

\[
v^2 = 240\left(\frac{1}{\sqrt{x}} - \frac{1}{2}\right)
\]

(a) When \( x = 2 \) m:

\[
v^2 = 240\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right)
\]

or

\( v = -7.05 \) m/s

(b) When \( x = 1 \) m:

\[
v^2 = 240\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right)
\]

or

\( v = -10.95 \) m/s

(c) When \( x = 0.1 \) m:

\[
v^2 = 240\left(\frac{1}{\sqrt{0.1}} - \frac{1}{2}\right)
\]

or

\( v = -25.3 \) m/s
PROBLEM 11.184
A projectile enters a resisting medium at \( x = 0 \) with an initial velocity \( v_0 = 270 \, \text{m/s} \) and travels 100 mm before coming to rest. Assuming that the velocity of the projectile is defined by the relation \( v = v_0 - kx \), where \( v \) is expressed in m/s and \( x \) is in m, determine (a) the initial acceleration of the projectile, (b) the time required for the projectile to penetrate 97.5 mm into the resisting medium.

SOLUTION
First note
When \( x = 0.1 \, \text{m}, v = 0 \):
\[
0 = (270 \, \text{m/s}) - k(0.1 \, \text{m})
\]
or
\[
k = 2700 \, \text{s}^{-1}
\]
(a) We have
\[
v = v_0 - kx
\]
Then
\[
a = \frac{dv}{dt} = \frac{d}{dt}(v_0 - kx) = -kv
\]
or
\[
a = -k(v_0 - kx)
\]
At \( t = 0 \):
\[
a = -2700 \left(\frac{1}{s}\right)(270 \, \text{m/s} - 0)
\]
or
\[
a_0 = -729 \times 10^3 \, \text{m/s}^2
\]
(b) We have
\[
\frac{dx}{dt} = v = v_0 - kx
\]
At \( t = 0, x = 0 \):
\[
\int_0^x \frac{dx}{v_0 - kx} = \int_0^t dt
\]
or
\[
-\frac{1}{k}[\ln(v_0 - kx)]_0^x = t
\]
or
\[
t = \frac{1}{k} \ln \left( \frac{v_0}{v_0 - kx} \right) = \frac{1}{k} \ln \left( \frac{1}{1 - \frac{k}{v_0} x} \right)
\]
When \( x = 97.5 \, \text{mm} = 0.0975 \, \text{m} \)
\[
t = \frac{1}{2700 \, \text{s}^{-1}} \ln \left[ \frac{1}{1 - \frac{2700 \, \text{s}^{-1}}{2700 \, \text{m/s}} \left(0.0975 \, \text{m}\right)} \right]
\]
or
\[
t = 1.366 \times 10^{-3} \, \text{s}
\]
**PROBLEM 11.185**

A freight elevator moving upward with a constant velocity of 2 m/s passes a passenger elevator which is stopped. Four seconds later, the passenger elevator starts upward with a constant acceleration of 0.8 m/s\(^2\). Determine (a) when and where the elevators will be at the same height, (b) the speed of the passenger elevator at that time.

**SOLUTION**

(a) For \( t \geq 0 \):

\[
y_F = 0 + v_F t
\]

\( t \geq 4 \text{ s} \):

\[
y_F = 0 + 0(t - 4) + \frac{1}{2}a_p(t - 4)^2
\]

When

\[
y_F = y_P
\]

\[
(2 \text{ m/s})t = \frac{1}{2}(0.8 \text{ m/s}^2)(t - 4)^2
\]

Expanding and simplifying

\[
t^2 - 13 + 16 = 0
\]

Solving

\[
t = 1.3765 \text{ s and } t = 11.6235 \text{ s}
\]

Must require \( t > 4 \) s

At \( t = 11.6235 \text{ s} \):

\[
y_F = (2 \text{ m/s})(11.6235 \text{ s}) = 23.247 \text{ m}
\]

or

\[
y_F = y_P = 23.2 \text{ m}
\]

(b) For \( t \geq 4 \text{ s} \):

\[
v_P = 0 + a_p(t - 4)
\]

At \( t = 11.6235 \text{ s} \):

\[
v_P = (0.8 \text{ m/s}^2)(11.6235 - 4) \text{ s}
\]

or

\[
v_P = 6.10 \text{ m/s}
\]
**PROBLEM 11.186**

Block C starts from rest at \( t = 0 \) and moves upward with a constant acceleration of 25 mm/s². Knowing that block A moves downward with a constant velocity of 75 mm/s, determine (a) the time for which the velocity of block B is zero, (b) the corresponding position of block B.

**SOLUTION**

The cable lengths are constant.

\[
L_1 = 2y_C + 2y_D + \text{constant}
\]
\[
L_2 = y_A + y_B + (y_B - y_D) + \text{constant}
\]

Eliminate \( y_D \).

\[
L_1 + 2L_2 = 2y_C + 2y_B + 2y_A + 2y_B + 2(y_B - y_D) + \text{constant}
\]
\[
2(y_C + y_A + 2y_B) = \text{constant}
\]

Differentiate to obtain relationships for velocities and accelerations, positive downward.

\[
v_C + v_A + 2v_B = 0 \quad (1)
\]
\[
a_C + a_A + 2a_B = 0 \quad (2)
\]

**Initial velocities.**

\[
(v_C)_0 = 0, \quad (v_A)_0 = 75 \text{ mm/s}
\]

From (1),

\[
(v_B)_0 = -\frac{1}{2}[(v_C)_0 + (v_A)_0] = -37.5 \text{ mm/s}
\]

**Accelerations.**

\[
a_C = -25 \text{ mm/s}^2 \quad a_A = 0
\]

From (2),

\[
a_B = -\frac{1}{2}(a_C + a_A) = 12.5 \text{ mm/s}^2
\]

**Motion of block B.**

\[
v_B = (v_B)_0 + a_B t = -37.5 + 12.5t
\]
\[
\Delta y_B = (v_B)_0 t + \frac{1}{2}a_B t^2 = -37.5t + 6.25t^2
\]

(a) **Time at \( v_B = 0 \).**

\[-37.5 + 12.5t = 0 \quad t = 3.00 \text{ s} \]

(b) **Corresponding position.**

\[
\Delta y_B = (-37.5)(3.00) + (6.25)(3.00)^2 \quad \Delta y_B = -56.25 \text{ mm}
\]
PROBLEM 11.187

The three blocks shown move with constant velocities. Find the velocity of each block, knowing that the relative velocity of A with respect to C is 300 mm/s upward and that the relative velocity of B with respect to A is 200 mm/s downward.

SOLUTION

From the diagram

Cable 1: \[ y_A + y_D = \text{constant} \]
Then \[ v_A + v_D = 0 \] (1)

Cable 2: \[ (y_B - y_D) + (y_C - y_D) = \text{constant} \]
Then \[ v_B + v_C - 2v_D = 0 \] (2)

Combining Eqs. (1) and (2) to eliminate \( v_D \)
\[ 2v_A + v_B + v_C = 0 \] (3)

Now \[ v_{AC} = v_A - v_C = -300 \text{ mm/s} \] (4)
and \[ v_{BA} = v_B - v_A = 200 \text{ mm/s} \] (5)
Then 
\[ (3) + (4) - (5) \Rightarrow \]
\[ (2v_A + v_B + v_C) + (v_A - v_C) - (v_B - v_A) = (-300) - (200) \]
or \[ v_A = 125 \text{ mm/s} \uparrow \]
and using Eq. (5) \[ v_B - (-125) = 200 \]
or \[ v_B = 75 \text{ mm/s} \downarrow \]
Eq. (4) \[ -125 - v_C = -300 \]
or \[ v_C = 175 \text{ mm/s} \downarrow \]
**PROBLEM 11.188**

An oscillating water sprinkler at Point A rests on an incline which forms an angle $\alpha$ with the horizontal. The sprinkler discharges water with an initial velocity $v_0$ at an angle $\phi$ with the vertical which varies from $-\phi_0$ to $+\phi_0$. Knowing that $v_0 = 10 \text{ m/s}$ and $\phi_0 = 40^\circ$, and $\alpha = 10^\circ$, determine the horizontal distance between the sprinkler and Points B and C which define the watered area.

**SOLUTION**

First note

$$(v_0)_x = v_0 \sin \phi = (10 \text{ m/s}) \sin \phi$$

$$(v_0)_y = v_0 \cos \phi = (10 \text{ m/s}) \cos \phi$$

Also, along incline CAB

$$y = x \tan 10^\circ$$

**Horizontal motion. (Uniform)**

$$x = 0 + (v_0)_x t = (10 \sin \phi) t \quad \text{or} \quad t = \frac{x}{10 \sin \phi}$$

**Vertical motion. (Uniformly accelerated motion)**

$$y = 0 + (v_0)_y t - \frac{1}{2} gt^2 = (10 \cos \phi) t - \frac{1}{2} gt^2$$

Substituting for $t$

$$y = (10 \cos \phi) \left( \frac{x}{10 \sin \phi} \right) - \frac{1}{2} g \left( \frac{x}{10 \sin \phi} \right)^2$$

$$= \frac{x}{\tan \phi} - \frac{9.81}{200} \frac{x^2}{\sin^2 \phi} \quad (g = 9.81 \text{ m/s}^2)$$

At B: $\phi = 40^\circ$, $x = d_B$

$$d_B \tan 10^\circ = \frac{d_B}{\tan 40^\circ} - \frac{9.81}{200} \frac{d_B^2}{\sin^2 40^\circ}$$

$$d_B = 8.55 \text{ m}$$

At C: $\phi = -40^\circ$, $x = -d_C$

$$-d_C \tan 10^\circ = \frac{-d_C}{\tan (-40^\circ)} - \frac{9.81}{200} \frac{(-d_C)^2}{\sin^2 (-40^\circ)}$$

$$d_C = 11.52 \text{ m}$$
**PROBLEM 11.189**

As the driver of an automobile travels north at 25 km/h in a parking lot, he observes a truck approaching from the northwest. After he reduces his speed to 15 km/h and turns so that he is traveling in a northwest direction, the truck appears to be approaching from the west. Assuming that the velocity of the truck is constant during the period of observation, determine the magnitude and the direction of the velocity of the truck.

**SOLUTION**

We have \( \mathbf{v}_T = \mathbf{v}_A + \mathbf{v}_T/A \)

Using this equation, the two cases are then graphically represented as shown.

From the diagram

\[
(v_T)_x = 25 - 15\sin 45^\circ \\
= 14.3934 \text{ km/h} \\
(v_T)_y = 15\sin 45^\circ \\
= 10.6066 \text{ km/h}
\]

\[ \mathbf{v}_T = 17.88 \text{ km/h} \hat{\theta} 36.4^\circ \]
PROBLEM 11.190

The driver of an automobile decreases her speed at a constant rate from 70 to 45 km/h over a distance of 225 m along a curve of 450 m radius. Determine the magnitude of the total acceleration of the automobile after the automobile has traveled 150 m along the curve.

SOLUTION

First note

\[ v_1 = 70 \text{ km/h} = \frac{175}{9} \text{ m/s} \]
\[ v_2 = 45 \text{ km/h} = \frac{25}{2} \text{ m/s} \]

We have uniformly decelerated motion

\[ v^2 = v_1^2 + 2a(s - s_1) \]

When \( v = v_2 \):

\[ \left( \frac{25}{2} \text{ m/s} \right)^2 = \left( \frac{175}{9} \text{ m/s} \right)^2 + 2a(225 \text{ m}) \]

or

\[ a = -0.49297 \text{ m/s}^2 \]

Then when \( \Delta s = 150 \text{ m} \)

\[ v^2 = \left( \frac{175}{9} \text{ m/s} \right)^2 + 2(-0.49297 \text{ m/s}^2)(150 \text{ m}) \]
\[ = 230.195 \text{ m}^2/\text{s}^2 \]

Now

\[ a_n = \frac{v^2}{\rho} \]
\[ = \frac{230.195 \text{ m}^2/\text{s}^2}{450 \text{ m}} \]
\[ = 0.51154 \text{ m/s}^2 \]

Finally

\[ a^2 = a_n^2 + a^2 \]
\[ = (-0.49297 \text{ m/s}^2)^2 + (0.51154 \text{ m/s}^2)^2 \]

or

\[ a = 0.710 \text{ m/s}^2 \]
PROBLEM 11.191

A homeowner uses a snowblower to clear his driveway. Knowing that the snow is discharged at an average angle of 40° with the horizontal, determine the initial velocity \( v_0 \) of the snow.

SOLUTION

First note

\[
(v_x)_0 = v_0 \cos 40° \\
(v_y)_0 = v_0 \sin 40°
\]

Horizontal motion. (Uniform)

\[
x = 0 + (v_x)_0 t
\]

At B:

\[
4.2 = (v_0 \cos 40°) t \quad \text{or} \quad t_B = \frac{4.2}{v_0 \cos 40°}
\]

Vertical motion. (Uniformly accelerated motion)

\[
y = 0 + (v_y)_0 t - \frac{1}{2} gt^2
\]

At B:

\[
0.45 = (v_0 \sin 40°) t_B - \frac{1}{2} gt_B^2
\]

Substituting for \( t_B \)

\[
0.45 = (v_0 \sin 40°) \left( \frac{4.2}{v_0 \cos 40°} \right) - \frac{1}{2} g \left( \frac{4.2}{v_0 \cos 40°} \right)^2
\]

or

\[
v_0^2 = \frac{\frac{1}{2} (9.81)(17.64)/\cos^2 40°}{-0.45 + 4.2\tan 40°}
\]

or

\[
v_0 = 6.93 \text{ m/s}
\]
PROBLEM 11.192

From measurements of a photograph, it has been found that as the stream of water shown left the nozzle at A, it had a radius of curvature of 25 m. Determine (a) the initial velocity \( v_A \) of the stream, (b) the radius of curvature of the stream as it reaches its maximum height at B.

SOLUTION

(a) We have \( (a_n)_n = \frac{v_A^2}{\rho_A} \)

or

\[ v_A^2 = \left( \frac{4}{5} \times 9.81 \text{ m/s}^2 \right) (25 \text{ m}) \]

or

\[ v_A = 14.0071 \text{ m/s} \]

\( v_A = 14.01 \text{ m/s} \) at \( 36.9^\circ \)

(b) We have \( (a_B)_n = \frac{v_B^2}{\rho_B} \)

where

\[ v_B = (v_A)_x = \frac{4}{5} v_A \]

Then

\[ \rho_B = \frac{\left( \frac{4}{5} \times 14.0071 \text{ m/s} \right)^2}{9.81 \text{ m/s}^2} \]

or

\( \rho_B = 12.80 \text{ m} \)
**PROBLEM 11.193**

At the bottom of a loop in the vertical plane, an airplane has a horizontal velocity of 150 m/s and is speeding up at a rate of 25 m/s². The radius of curvature of the loop is 2000 m. The plane is being tracked by radar at O. What are the recorded values of \( \dot{r} \), \( \dot{\theta} \), \( \theta \) and \( \dot{\theta} \) for this instant?

**SOLUTION**

**Geometry.** The polar coordinates are

\[
\begin{align*}
\theta &= \tan^{-1}\left(\frac{600}{800}\right) = 36.87° \\
v &= \sqrt{(800)^2 + (600)^2} = 1000 \text{ m/s}
\end{align*}
\]

**Velocity Analysis.**

\[
\begin{align*}
v &= 150 \text{ m/s} \\
v_r &= 150 \cos \theta = 120 \text{ m/s} \\
v_\theta &= -150 \sin \theta = -90 \text{ m/s} \\
r &= 120 \text{ m/s} \\
\dot{\theta} &= \frac{v_\theta}{r} = \frac{-90}{1000} = -0.0900 \text{ rad/s}
\end{align*}
\]

**Acceleration Analysis.**

\[
\begin{align*}
a_r &= 25 \text{ m/s}^2 \\
a_\theta &= \frac{v^2}{\rho} = \frac{(150)^2}{2000} = 11.25 \text{ m/s}^2 \\
a &= 25 \text{ m/s}^2 \rightarrow +11.25 \text{ m/s}^2 \uparrow = 27.41 \text{ m/s}^2 \uparrow^\circ 24.23° \\
\beta &= 24.23° \\
\theta - \beta &= 12.64°
\end{align*}
\]
PROBLEM 11.193 (Continued)

\[ a_r = \cos (\theta - \beta) = 27.41 \cos 12.64^\circ = 26.74 \text{ m/s}^2 \]
\[ a_\theta = -\sin (\theta - \beta) = -27.41 \sin 12.64^\circ = -6.00 \text{ m/s}^2 \]
\[ a_r = r - r\dot{\theta}^2 \quad r = a_r + r\dot{\theta}^2 \]
\[ r = 26.74 + (1000)(0.0900)^2 \]
\[ a_\theta = r\ddot{\theta} + 2r\dot{\theta} \]
\[ \ddot{\theta} = \frac{a_\theta}{r} - \frac{2r\dot{\theta}}{r} \]
\[ = \frac{-6.00}{1000} - \frac{(2)(120)(-0.0900)}{1000} \]
\[ \ddot{\theta} = -0.0156 \text{ rad/s}^2 \]

\[ r = 34.8 \text{ m/s}^2 \]
\[ \ddot{\theta} = -0.0156 \text{ rad/s}^2 \]