SOLVED EXAMPLES

Example 1 The position and momentum of a 1.0 keV electron are simultaneously measured. If the position is located within 1Å, what is the percentage of uncertainty in momentum?

Solution Given $\Delta x = 1.0 \times 10^{-10}$m and $E = 1000 \times 1.6 \times 10^{-19}$J $= 1.6 \times 10^{-16}$ J.

Heisenberg's uncertainty principle says

$$\Delta x \Delta p = \frac{\hbar}{2} \quad \text{and} \quad p = \sqrt{2mE}$$

$$p = \sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-16}}$$

$$= 1.71 \times 10^{-23} \text{kg m/sec}$$

and

$$\Delta p = \frac{\hbar}{2\Delta x} = \frac{h}{2 \times 2\pi \times \Delta x} = \frac{6.62 \times 10^{-34}}{2 \times 2 \times 3.14 \times 1.0 \times 10^{-10}}$$

$$= 5.27 \times 10^{-25} \text{ kg m/sec}$$

Percentage of uncertainty in momentum

$$= \frac{\Delta p}{p} \times 100 = \frac{5.27 \times 10^{-25}}{1.71 \times 10^{-23}} \times 100$$

$$= \mathbf{3.1\%}$$

Example 2 The uncertainty in the location of a particle is equal to its deBroglie wavelength. Calculate the uncertainty in its velocity.

Solution Given $\Delta x = \frac{h}{p}$.

$$\Delta x\,\Delta p = \frac{\hbar}{2} = \frac{h}{4\pi}$$

or $$\Delta p = \Delta(mv) = \frac{h}{4\pi}\frac{1}{\Delta x} = \frac{h}{4\pi}\frac{p}{h} = \frac{mv}{4\pi}$$

$$m\Delta v = \frac{mv}{4\pi}$$

or $$\Delta v = \frac{v}{4\pi}$$

Example 3 The position and momentum of 0.5 keV electron are simultaneously determined. If its position is located within 0.2 nm, what is the percentage uncertainty in its momentum?

Solution Given $E = 0.5 \times 10^{3} \times 1.6 \times 10^{-19} = 0.8 \times 10^{-16}$ J and $\Delta x = 0.2 \times 10^{-9}$ m.

Now

$$\Delta x\Delta p = \frac{\hbar}{2} \text{ and momentum } p = \sqrt{2mE}$$

so $$p = \sqrt{2\times 9.1\times 10^{-31}\times 0.8\times 10^{-16}} = 12.06\times 10^{-24}$$

or $$p = 1.21\times 10^{-23} \text{ kg m/sec}$$

or $$\Delta p = \frac{\hbar}{2}\frac{1}{\Delta x} = \frac{h}{4\pi}\frac{1}{0.2\times 10^{-9}}$$

$$\Delta p = \frac{6.62\times 10^{-34}}{4\times 3.14\times 0.2\times 10^{-9}} = 2.635\times 10^{-25} \text{kg m/sec}$$

Percentage uncertainty in momentum

$$\frac{\Delta p}{p}\times 100 = \frac{2.635\times 10^{-25}}{1.21\times 10^{-23}}\times 100$$

$$= \frac{2.635\times 10^{-23}}{1.21\times 10^{-23}} = \mathbf{2.18\%}$$

Example 4 Wavelengths can be determined with accuracies of one part in 10^6. What is the uncertainty in the position of a 1Å X-ray photon when its wavelength is simultaneously measured?

Solution Given $\lambda = 10^{-10}$ m.

By uncertainty principle,

$$\therefore \quad \Delta x\Delta p = \frac{\hbar}{2} = \frac{h}{4\pi} \quad \text{(i)}$$

and $$\lambda = \frac{h}{p} \quad \text{or} \quad p\lambda = h$$

By differentiating

$$p\Delta\lambda + \lambda\Delta p = 0$$

$$\text{or} \quad \Delta p = -\frac{p\Delta\lambda}{\lambda} = -\frac{h\Delta\lambda}{\lambda^2} \qquad \left[\because p = \frac{h}{\lambda}\right] \tag{ii}$$

By using Eqs. (i) and (ii), we get

$$\Delta x \frac{h\Delta\lambda}{\lambda^2} = \frac{h}{4\pi}$$

$$\text{or} \quad \Delta x \Delta\lambda = \frac{\lambda^2}{4\pi} \tag{iii}$$

Wavelength can be measured with accuracy of one part in 10^6, it means the uncertainty in wavelength is

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{10^6} = 10^{-6} \tag{iv}$$

By putting this value in Eq. (iii), then

$$\Delta x \frac{\Delta\lambda}{\lambda} = \frac{\lambda}{4\pi} \quad \text{or} \quad \Delta x \times 10^{-6} = \frac{\lambda}{4\pi}$$

$$\text{or} \quad \Delta x = \frac{10^6 \times \lambda}{4\pi} = \frac{10^6 \times 10^{-10}}{4 \times 3.14} = \mathbf{7.96\ \mu m}$$

Example 5 Calculate the uncertainty in measurement of momentum of an electron if the uncertainty in locating it is 1Å.

Solution Given $\Delta x = 1.0 \times 10^{-10}$ m.

Formula used is

$$\Delta x \Delta p = \frac{\hbar}{2} = \frac{h}{4\pi}$$

$$\Delta p = \frac{h}{4\pi}\frac{1}{\Delta x} = \frac{6.62 \times 10^{-34}}{4 \times 3.14} \times \frac{1}{10^{-10}}$$

$$\Delta p = \mathbf{5.27 \times 10^{-25}\ kg\ m/sec}$$

Example 6 An electron has a momentum 5.4×10^{-26} kg m/sec with an accuracy of 0.05%. Find the minimum uncertainty in the location of the electron.

Solution Given $p = 5.4 \times 10^{-26}$ kg m/sec.

The uncertainty in the measurement of momentum

$$\Delta p = \frac{5.4 \times 10^{-26} \times 0.05}{100}$$

$$= 2.7 \times 10^{-29}\ \text{kg m/sec}$$

$$\Delta x \Delta p = \frac{\hbar}{2} = \frac{h}{4\pi}$$

$$\therefore \ \Delta x = \frac{h}{4\pi}\frac{1}{\Delta p} = \frac{6.62\times10^{-34}}{4\times3.14}\times\frac{1}{2.7\times10^{-29}}$$
$$= 1.952\times10^{-6}\,\text{m}$$
$$= \mathbf{1.952\ \mu m}$$

(ii)

Example 7 A hydrogen atom is 0.53 Å in radius. Use uncertainty principle to estimate the minimum energy an electron can have in this atom.

(iii)

Solution Given $\Delta x_{max} = 0.53\text{Å}$.

Heisenberg's uncertainty principle

$$\Delta x \Delta p = \frac{\hbar}{2} = \frac{h}{4\pi}$$

(iv)

$$(\Delta x)_{max}(\Delta p)_{min} = \frac{h}{4\pi} \qquad \text{(i)}$$

$$\text{and}\quad (\text{K.E.})_{min} = \frac{p^2_{\ min}}{2m} = \frac{(\Delta p)^2_{\ min}}{2m} \qquad [\because p_{min} = \Delta p_{min}]$$

$$(\Delta p)_{min} = \frac{h}{4\pi}\frac{1}{\Delta x} = \frac{6.62\times10^{-34}}{4\times3.14}\frac{1}{0.53\times10^{-10}}$$
$$= 0.9945\times10^{-24}$$
$$= 9.945\times10^{-25}\,\text{kg m/sec}$$

$$\text{and}\quad (\text{K.E.})_{min} = \frac{(\Delta p)^2_{\ min}}{2m} = \frac{(9.945\times10^{-24})^2}{2\times9.1\times10^{-31}}$$
$$= \mathbf{5.434\times10^{-17}\,J}$$

Example 8 The speed of an electron is measured to be 5.0×10^3 m/sec to an accuracy of 0.003%. Find the uncertainty in determining the position of this electron.

Solution Given $v = 5.0\times10^3$ m/sec.

Formula used is

$$\Delta x \Delta p = \frac{\hbar}{2} = \frac{h}{4\pi}$$

$$\Delta v = v\times\frac{0.003}{100} = 5.0\times10^3\times\frac{0.003}{100} = 0.15\,\text{m/sec}$$

$$\text{and}\quad \Delta p = m\Delta v = 9.1\times10^{-31}\times0.15 = 1.365\times10^{-31}\,\text{kg m/sec}$$

$$\Delta x = \frac{6.62\times10^{-34}}{4\times3.14}\frac{1}{\Delta p} = \frac{6.62\times10^{-34}}{4\times3.14}\frac{1}{1.365\times10^{-31}}$$
$$= \mathbf{3.861\times10^{-4}\,m}$$

Example 9 An electron has speed of 6.6×10^4 m/sec with an accuracy of 0.01%. Calculate the uncertainty in position of an electron. Given mass of an electron as 9.1×10^{-31} kg and Planck's constant h as 6.6×10^{-34} J sec.

Solution Given $v = 6.6\times10^4$ m/sec and $\Delta v = 6.6\times10^4\times\frac{0.01}{100}$ m/sec

$$= 6.6\ \text{m/sec.}$$

Formula used is

$$\Delta x\Delta p=\frac{\hbar}{2}=\frac{h}{4\pi} \quad \text{or} \quad \Delta x=\frac{h}{4\pi}\frac{1}{\Delta p}$$

$$\Delta p=m\Delta v=9.1\times10^{-31}\times6.6$$

$$\text{or}\quad \Delta x=\frac{h}{4\pi}\frac{1}{\Delta p}=\frac{6.6\times10^{-34}}{4\times3.14\times9.1\times10^{-31}\times6.6}$$

$$\Delta x=\mathbf{8.75\times10^{-6}\,m}$$

Example 10 Calculate the smallest possible uncertainty in the position of an electron moving with a velocity 3×10^7 m/sec.

Solution Given $v=3\times10^7$ m/sec.

Formula used is

$$\Delta x\Delta p=\frac{\hbar}{2}=\frac{h}{4\pi}$$

$$\Delta p_{\min}\approx p=mv=\frac{m_0 v}{\sqrt{1-v^2/c^2}}$$

$$\therefore \quad \Delta x=\frac{h}{4\pi}\frac{1}{\Delta p}=\frac{h}{4\pi}\left[\frac{\sqrt{1-v^2/c^2}}{m_0 v}\right]$$

$$=\frac{6.62\times10^{-34}}{4\times3.14}\left[\frac{\sqrt{1-\left(\frac{3\times10^7}{3\times10^8}\right)^2}}{9.1\times10^{-31}\times3\times10^7}\right]$$

$$=\mathbf{1.92\times10^{-12}\,m}$$

Example 11 If an excited state of hydrogen atom has a life-time of 2.5×10^{-14} sec, what is the minimum error with which the energy of this state can be measured? Given $h=6.62\times10^{-34}$ J sec.

Solution Given $\Delta t=2.5\times10^{-14}$ sec.

Formula used is

$$\Delta E\Delta t=\frac{\hbar}{2}=\frac{h}{4\pi}$$

$$\Delta E=\frac{h}{4\pi}\frac{1}{\Delta t}=\frac{6.62\times10^{-34}}{4\times3.14}\times\frac{1}{2.5\times10^{-14}}=0.211\times10^{-20}\,J$$

$$\Delta E=\mathbf{2.11\times10^{-21}\,J}$$

Example 12 An excited atom has an average life-time of 10^{-2} sec. During this time period it emits a photon and returns to the ground state. What is the minimum uncertainty in the frequency of this photon?

Solution Given $\Delta t=10^{-8}$ sec.

Example 29 Calculate the deBroglie wavelength associated with the automobile of mass 2×10^3 kg which is moving with a speed 96 km/hr.

Solution Given $m = 2\times10^3\text{kg},\ v = \dfrac{96\times10^3}{60\times60}$ m/sec $= 26.67$ m/sec.

deBroglie wavelength is given as

$$\lambda = \frac{h}{mv} = \frac{6.62\times10^{-34}}{2\times10^3\times26.67}$$
$$= 0.124\times10^{-37}\text{m}$$
$$= \mathbf{1.24\times10^{-38}\,m}$$

Example 30 A particle of charge q and mass m is accelerated through a potential difference V. Find its deBroglie wavelength. Calculate the wavelength (λ), if the particle is an electron and V = 50 volts.

Solution When a particle of charge q and mass m is accelerated through a potential V, then deBroglie wavelength is given by

$$\lambda = \frac{h}{mv} \quad \text{(i)}$$

and $E_k = \dfrac{1}{2}mv^2 = qV$ or $m^2v^2 = 2mqV$

or $mv = \sqrt{2mqV}$ (ii)

By using Eqs. (i) and (ii), we obtain

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

Given $q = 1.6 \times 10^{-19}$ C and V = 50 volts, then

$$\lambda = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times50}}$$
$$= \mathbf{1.74\ \text{Å}}$$

Example 31 Calculate the wavelength of thermal neutrons at 27°C, given mass of neutron = 1.67×10^{-27}kg, Planck's constant $h = 6.6 \times 10^{-34}$ J·s and Boltzmann's constant $k = 1.37\times10^{-23}\text{JK}^{-1}$.

Solution Given T = 27°C = 27 + 273 = 300K, m = 1.67×10^{-27}kg, h = 6.6×10^{-34} Jsec and $k = 1.376 \times 10^{-23}\text{JK}^{-1}$.

deBroglie wavelength is given by

$$\lambda = \frac{h}{mv} \quad \text{(i)}$$

$E_k = \dfrac{1}{2}mv^2 = \dfrac{3}{2}kT$ or $(mv)^2 = 3mkT$ (ii)

or $mv = \sqrt{3mkT}$

Then, $\lambda = \frac{h}{\sqrt{3mkT}} = \frac{6.6\times10^{-34}}{\sqrt{3\times1.67\times10^{-27}\times1.376\times10^{-23}\times300}}$

$= 1.452\times10^{-10}\text{m}$

or $\lambda = \mathbf{1.452Å}$

Example 32 A proton is moving with a speed 2×10^8 m/sec. Find the wavelength of matter wave associ[ated] with it.

Solution Given $v = 2\times10^8$ m/sec.

Formula used for deBroglie wavelength is

$$\lambda = \frac{h}{mv} = \frac{6.62\times10^{-34}}{1.67\times10^{-27}\times2\times10^8}$$
$$= \mathbf{1.98\times10^{-15}m}$$

Example 33 The deBroglie wavelength associated with an electron is 0.1Å. Find the potential difference by which the electron is accelerated.

Solution Given $\lambda = 0.1\times10^{-10}$ m.

deBroglie wavelength in terms of potential difference is given by

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

or $2mqV = \frac{h^2}{\lambda^2}$

or $V = \frac{h^2}{2mq\lambda^2} = \frac{(6.62\times10^{-34})^2}{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times(10^{-11})^2}$

$= \mathbf{15.05kV}$

Example 34 Calculate the deBroglie wavelength of an α-particle accelerated through a potential difference of 200 volts.

Solution Given V = 200 volts, $q = q_\alpha = 2e = 3.2\times10^{-19}$ C and $m = m_\alpha = 4m_p$.

deBroglie wavelength in terms of potential difference

$$\lambda = \frac{h}{\sqrt{2m_\alpha qV}} = \frac{6.62\times10^{-34}}{\sqrt{2\times4\times1.67\times10^{27}\times2\times1.6\times10^{-19}\times200}}$$
$$= \frac{6.62\times10^{-34}}{92.468\times10^{-23}} = 0.07159\times10^{-11}$$
$$\lambda = \mathbf{7.16\times10^{-13}m}$$

Example 35 Calculate the deBroglie wavelength of an average Helium atom in furnace of 400 K. Given $k = 1.38\times10^{-23}$ J/K

Solution Given T = 400 K, $k = 1.38\times10^{-23}$ J/K and mass of Helium atom = $4m_p = 4 \times 1.67 \times 10^{-27}$ kg.

deBroglie wavelength in terms of temperature i.e.

$$\lambda = \frac{h}{\sqrt{3mkT}} = \frac{6.62\times10^{-34}}{\sqrt{3\times4\times1.67\times10^{-27}\times1.38\times10^{-23}\times400}}$$
$$= \frac{6.62\times10^{-34}}{105.176\times10^{-25}} = 0.6294Å$$
$$\lambda = \mathbf{0.6294Å}$$

Example 36 Calculate the deBroglie wavelength associated with a neutron moving with a velocity of 2000 m/sec.

Solution Given v = 2000 m/sec and $m = 1.67 \times 10^{-27}$ kg.

deBroglie wavelength

$$\lambda = \frac{h}{mv} = \frac{6.62\times10^{-34}}{1.67\times10^{-27}\times2000}$$
$$= 1.98\times10^{-10}\text{m}$$
$$= \mathbf{1.98Å}$$

Example 37 Calculate the energy in eV corresponding to a wavelength of 1.0 Å for electron and neutron. Given $h = 6.6 \times 10^{-34}$ J sec, mass of electron = 9.1×10^{-31} kg and mass of the neutron = 1.7×10^{-27} kg.

Solution

Formula used is

$$\lambda = \frac{h}{mv} \quad \text{or} \quad v = \frac{h}{\lambda m}$$
$$\text{or} \quad v = \frac{6.6\times10^{-34}}{1.0\times10^{-10}\times1.7\times10^{-27}}$$
$$= \mathbf{3.88\times10^{3}\,m/sec}$$

If the velocity is much less than the velocity of light, it can be considered as non-relativistic case and hence deBroglie wavelength can be obtained by the relation.

$$\lambda = \frac{h}{\sqrt{2mE}} \quad \text{or} \quad \lambda^2 = \frac{h^2}{2mE}$$

For Electron

$$\text{or} \quad E = \frac{h^2}{2m\lambda^2} = \frac{(6.62\times10^{-34})^2}{2\times9.1\times10^{-31}\times(10^{-10})^2}$$
$$= \frac{43.8244\times10^{-68}}{18.2\times10^{-51}} = 2.41\times10^{-17}\text{J}$$
$$= 1.51\times100 = 151\text{eV}$$
$$\mathbf{E = 151eV}$$

neutron

$$E = \frac{(6.62\times10^{-34})^2}{2\times1.7\times10^{-27}\times(10^{-10})^2} = \frac{43.8244\times10^{-68}}{3.4\times10^{-47}}$$

$$= 12.89\times10^{-21}\text{J}$$

$$= \mathbf{0.081\ eV}$$

xample 38 Calculate deBroglie wavelength of an electron whose kinetic energy is (i) 500 eV, (ii) 50 eV and (iii) 1.0 eV.

Solution Formula used is

$$\lambda = \frac{h}{\sqrt{2mE}}$$

(i) $E = 500\,\text{eV} = 500\times1.6\times10^{-19} = 8.0\times10^{-17}\,\text{J}$

$$\lambda = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times8.0\times10^{-17}}} = 5.486\times10^{-11}\text{m}$$

$$= \mathbf{0.5486Å}$$

(ii) $E = 50\,\text{eV} = 50\times1.6\times10^{-19} = 8.0\times10^{-18}$

$$\lambda = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times8\times10^{-18}}}$$

$$\lambda = 1.735\times10^{-10}\text{m}$$

or $\lambda = \mathbf{1.735Å}$

(iii) $E = 1.0\,\text{eV} = 1.6\times10^{-19}\,\text{J}$

$$\lambda = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}}}$$

$$\lambda = \mathbf{12.267Å}$$

Example 39 Calculate the ratio of deBroglie wavelengths associated with the neutrons with kinetic energies of 1.0 eV and 510 eV.

Solution Formula used is

$$\lambda = \frac{h}{\sqrt{2mE}}.$$

For $E = 1.0\text{ eV} = 1.6\times10^{-19}$ J and $m_n = 1.7\times10^{-27}$ kg

$$\lambda_1 = \frac{6.62\times10^{-34}}{\sqrt{2\times1.7\times10^{-27}\times1.6\times10^{-19}}}$$

$$= 2.838\times10^{-11}$$

$$\lambda_1 = \mathbf{0.284Å}$$

For $E = 510\,\text{eV} = 510 \times 1.6 \times 10^{-19} = 816 \times 10^{-19}\,\text{J}$

$$\lambda_2 = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.7 \times 10^{-27} \times 816 \times 10^{-19}}}$$

$$\lambda_2 = 0.01257\,\text{Å}$$
$$= \mathbf{0.0126\,Å}$$

and ratio of deBroglie wavelength is

$$\frac{\lambda_1}{\lambda_2} = \frac{0.284}{0.0126} = \mathbf{22.54 : 1}$$

Example 40 Calculate the ratio of deBroglie waves associated with a proton and an electron each having the kinetic energy as 20 M eV [$m_p = 1.67 \times 10^{-27}$ kg and $m_e = 9.1 \times 10^{-31}$ kg].

Solution Given energy of each proton and electron is $20 \times 10^6 \times 10^{-19}\,\text{J} = 3.2 \times 10^{-12}\,\text{J}$.

Formula used is

$$\lambda = \frac{h}{\sqrt{2mE}}$$

For proton

$$\lambda_p = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 3.2 \times 10^{-12}}}$$
$$= 6.4 \times 10^{-15}\,\text{m}$$

For electron

$$\lambda_e = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 3.2 \times 10^{-12}}}$$
$$= 2.74 \times 10^{-13}\,\text{m}$$

The ratio of λ_p to λ_e is

$$\boldsymbol{\lambda_p : \lambda_e = 1:43}$$

Example 41 Calculate the deBroglie wavelength of 1.0 M eV proton. Do we require relativistic calculation?

Solution Given Energy $E = 1.0 \times 10^6 \times 1.6 \times 10^{-19}\,\text{J} = 1.6 \times 10^{-13}\,\text{J}$

Formula used for velocity of Proton

$$E = \frac{1}{2} m v^2 \quad \text{or} \quad v^2 = \frac{2E}{m}$$

$$\text{or} \quad v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-13}}{1.67 \times 10^{-27}}}$$
$$= \mathbf{1.38 \times 10^7\,m/sec}$$

From the above result it is clear that the velocity of proton is nearly one twentieth of the velocity of light. So the relativistic calculations are not required.

Example 42 Calculate the deBroglie wavelength associated with a proton moving with a velocity equal to $\frac{1}{20}^{th}$ of velocity of light.

Solution Given $v = \frac{c}{20} = \frac{3\times10^8}{20} = 1.5\times10^7$ m/sec and $m = 1.67\times10^{-27}$ kg.

Formula used is

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.62\times10^{-34}}{1.67\times10^{-27}\times1.5\times10^7}$$
$$= \mathbf{2.643\times10^{-14}\,m}$$

Example 43 Calculate the kinetic energy of a proton and an electron so that the deBroglie wavelengths associated with them is the same and equal to 5000 Å.

Solution Given wavelength of proton and electron = 5.0×10^{-7} m.

Formula used in

$$\lambda = \frac{h}{\sqrt{2mE}} \quad \text{or} \quad E = \frac{h^2}{2m\lambda^2}$$

For proton $m = m_p = 1.67 \times 10^{-27}$ kg and $\lambda = 5.0 \times 10^{-7}$ m

$$E = \frac{(6.62\times10^{-34})^2}{2\times1.67\times10^{-27}\times(5.0\times10^{-7})^2}$$
$$= \frac{43.8244\times10^{-68}}{83.5\times10^{-41}} = 0.5248\times10^{-27}\,J$$
$$= \mathbf{5.248\times10^{-28}\,J}$$

For electron $m = m_e = 9.1 \times 10^{-31}$

$$E = \frac{(6.62\times10^{-34})^2}{2\times9.1\times10^{-31}\times(5\times10^{-7})^2} = \frac{43.8244\times10^{-68}}{4.55\times10^{-43}}$$
$$E = \mathbf{9.63\times10^{-25}\,J}$$

Example 44 Find deBroglie wavelength of an electron in the first Bohr's orbit of hydrogen atom.

Solution Energy of an electron in the first Bohr's orbit of hydrogen atom can be obtained by using the relation

$$E_n = \frac{-13.6}{n^2}$$

$$E_1 = \frac{-13.6}{1^2} = -13.6\,eV$$
$$E_1 = -13.6\times1.6\times10^{-19}\,J = -2.176\times10^{-18}\,J$$

Magnitude of energy is $= 2.176\times10^{-18}$ J

$$\text{Wavelength } \lambda = \frac{h}{\sqrt{2mE}} = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times21.76\times10^{-19}}}$$
$$= 3.3\times10^{-10}\,m$$
$$= \mathbf{3.3\,Å}$$

Example 45 Calculate the ratio of deBroglie wavelengths of a hydrogen atom and helium atom at room temperature, when they move with thermal velocities. Given mass of hydrogen atom

$m_H = 1.67 \times 10^{-27}$kg and mass of helium atom $m_{He} = 4 \times m_p = 4 \times 1.67 \times 10^{-27}$kg at room temperature T = 27ºC = 300 K and Boltzmann's constant k = 1.376×10^{-23} J/K.

Solution deBroglie wavelength can be calculated by the relation

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

For Hydrogen atom

$$\lambda = \frac{6.62 \times 10^{-34}}{\sqrt{3 \times 1.67 \times 10^{-27} \times 1.376 \times 10^{-23} \times 300}}$$
$$= 1.456 \times 10^{-10} \text{m}$$
$$\lambda = \mathbf{1.456Å}$$

For Helium atom

$$\lambda_{He} = \frac{6.62 \times 10^{-34}}{\sqrt{3 \times 4 \times 1.67 \times 10^{-27} \times 1.376 \times 10^{-23} \times 300}}$$
$$= 0.728 \times 10^{-10} \text{m}$$
$$= \mathbf{0.728 Å}$$

The ratio of wavelengths i.e

$$\frac{\lambda_H}{\lambda_{He}} = \frac{1.456}{0.728} = \frac{2}{1}$$
$$\lambda_H : \lambda_{He} = 2:1$$

Example 46 A proton and a deuteron have the same kinetic energy. Which has a longer wavelength?

Solution m_p = mass of porton, $m_d = 2m_p$ and v_p and v_d are the velocities of proton and deuteron.

Kinetic energy of proton is given by

$$E_p = \frac{1}{2} m_p v_p^2$$

and kinetic energy of deuteron is

$$E_d = \frac{1}{2} m_d v_d^2 = \frac{1}{2}(2m_p)v_d^2$$
$$E_d = m_p v_d^2$$

But $E_p = E_d$, then

$$m_p v_d^2 = \frac{1}{2} m_p v_p^2$$

or $$v_d = \frac{v_p}{\sqrt{2}}$$

deBroglie wavelength corresponding to moving proton and deuteron are

$$\lambda_p = \frac{h}{m_p v_p} \quad \text{and}$$

$$\lambda_d = \frac{h}{m_d v_d} = \frac{h}{2m_p v_p / \sqrt{2}} = \frac{h}{\sqrt{2} m_p v_p}$$

$$\frac{\lambda_d}{\lambda_p} = \frac{h}{\sqrt{2}\, m_p v_p} \times \frac{m_p v_p}{h} = \frac{1}{\sqrt{2}}$$

$$\lambda_p = \sqrt{2}\, \lambda_d$$

i.e. proton has a longer wavelength.

Example 47 Find the phase and group velocities of an electron whose deBroglie wavelength is 1.2Å.

Solution Formula used is

$$\lambda = \frac{h}{mv}.$$

v_g = Group velocity = Particle velocity = v

$$v = \frac{h}{m\lambda} = \frac{6.62 \times 10^{-34}}{9.1 \times 10^{-31} \times 1.2 \times 10^{-10}}$$

$$v = 6.06 \times 10^{6}\ \text{m/sec} = \text{group velocity} = v_g$$

or $v_g = 6.06 \times 10^{6}$ m/sec

Phase velocity $v_p = \dfrac{\omega}{k}$

Energy $E = h\nu$

or $E = \dfrac{h}{2\pi} 2\pi\nu = \hbar\omega$

and momentum $p = \dfrac{h}{\lambda}$

or $P = \dfrac{h}{2\pi}\dfrac{2\pi}{\lambda} = \hbar k$

or $v_p = \dfrac{\omega}{k} = \dfrac{\hbar\omega}{\hbar k} = \dfrac{E}{p}$

and $E = \dfrac{1}{2} m v^2$ and $p = mv$

or $E = \dfrac{m^2 v^2}{2m} = \dfrac{p^2}{2m}$

$$v_p = \frac{E}{p}$$

$$v_p = \frac{p^2/2m}{p} = \frac{p}{2m} = \frac{h/\lambda}{2m} = \frac{h}{2m\lambda}$$

$$= \frac{6.62 \times 10^{-34}}{2 \times 9.1 \times 10^{-31} \times 1.2 \times 10^{-10}}$$

$$v_p = \mathbf{3.03 \times 10^{6}\ m/sec}$$

From the above result it is clear that the phase velocity is just half of group velocity.